Automatic Control (2)

By

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Lecture (3)

By

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Course Title: Automatic Control (2)

Course Code: EEC 415

Prerequisites: EEC224 Signals and Systems

Study Hours: 3 Cr. hrs.

= [2 Lect. + 0 Tut + 3 Lab]

Assessment:

Final Exam: 40%.

Midterm: 30%.

Quizzes: 10%.

Home assignments and Reports: 10%.

MATLAB Mini Project: 10%.

Textbook:

1- K. Ogata, Modern Control Engineering, Pearson, 5th. Ed., 2009.
2- Nise, N. S. "Control System Engineering", 7th edition, John Wiley & Sons Ltd., UK, 2016.
3- F. Golnaraghi and B. C. Kuo, "Automatic control Systems", 10th ed., John Wiley & Sons, Inc. 2017.
4- Andrea Bacciotti, "Stability and Control of Linear Systems" Volume 185, Springer, 2019.

Course Description

- Compensation in control systems, lead, lag, and lead-lag phase compensation in frequency domain,
- State model of linear systems using physical variables, state space representation using phase variables, state space representation, using canonical variables, properties of transition matrix and solution of state equation,
- Poles, zeros, eigen values and stability in multivariable system,
- Introduction to nonlinear control systems, describing function method, nature and stability of limit cycle.

Analysis & Design of Control Systems using Frequency Response

Relative Stability

A transfer function is called minimum phase when all the poles and zeroes are LHP and non-minimum-phase when there are RHP poles or zeroes.

Minimum phase system → Stable

The gain margin (GM) is the distance on the bode magnitude plot from the amplitude at the phase crossover frequency up to the 0 dB point. GM=-(dB of GH measured at the phase crossover frequency)

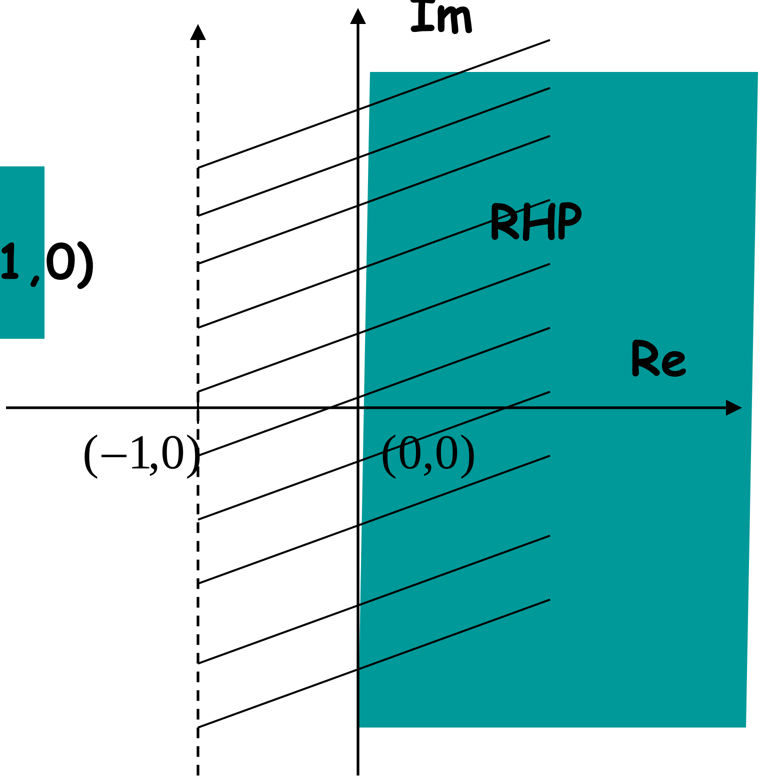
The phase margin (PM) is the distance from -180 up to the phase at the gain crossover frequency. PM=180+phase of GH measured at the gain crossover frequency

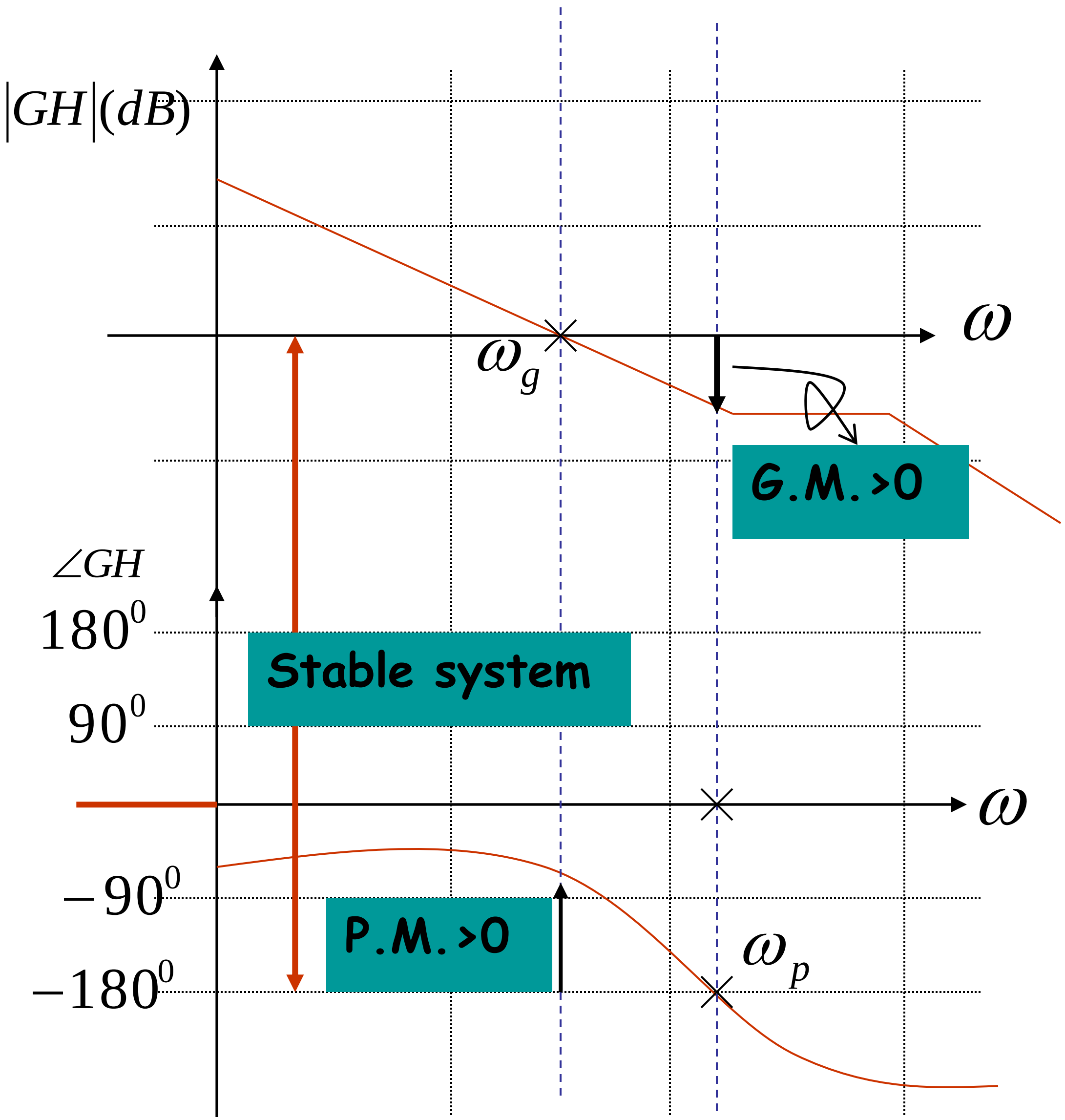
Open loop transfer function : $G(s)H(s)$

Closed-loop transfer function : $1+G(s)H(s)$

Open loop Stability → poles of $G(s)H(s)$ in LHP

Closed-loop Stability → poles of $G(s)H(s)$ in left side of (-1,0)

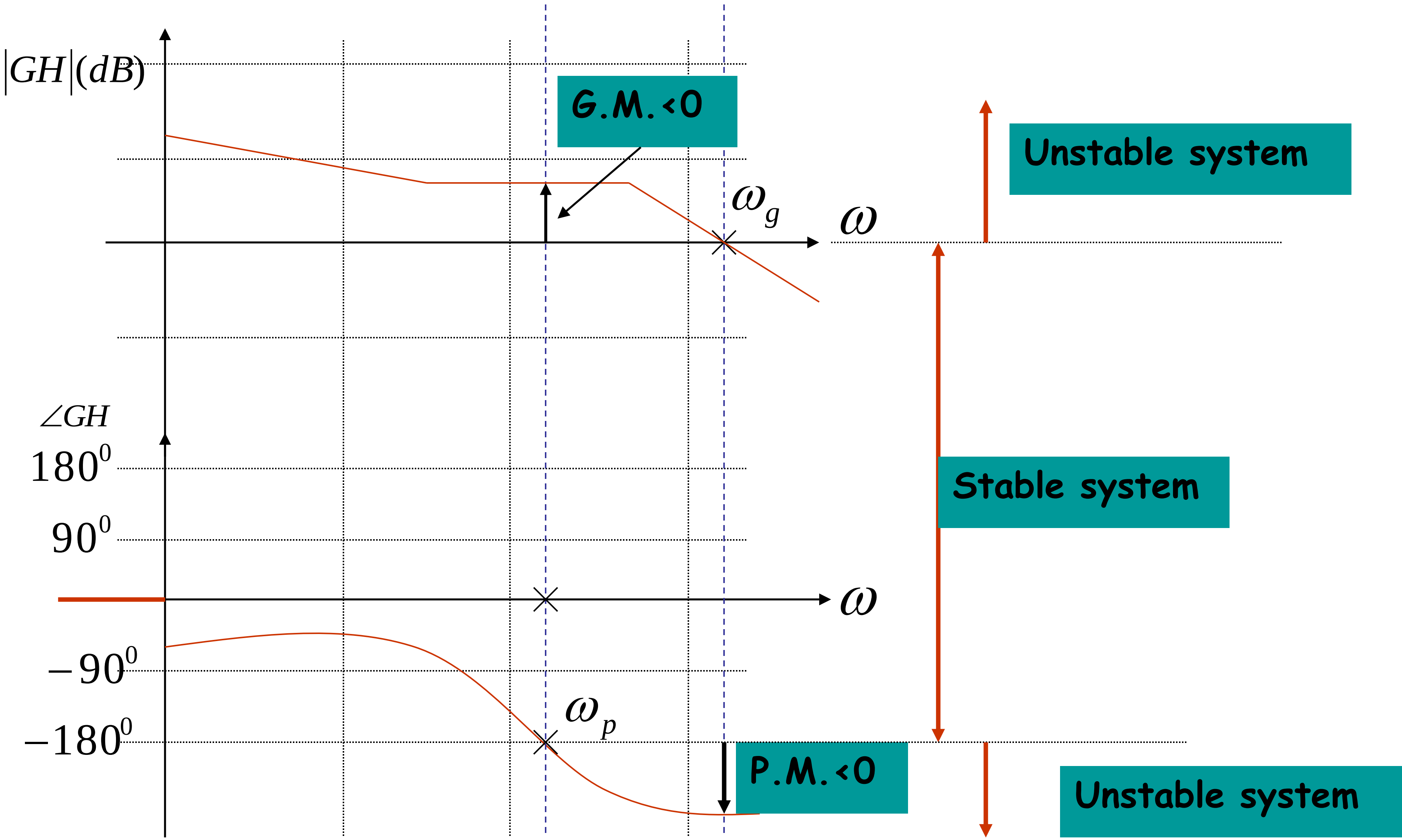




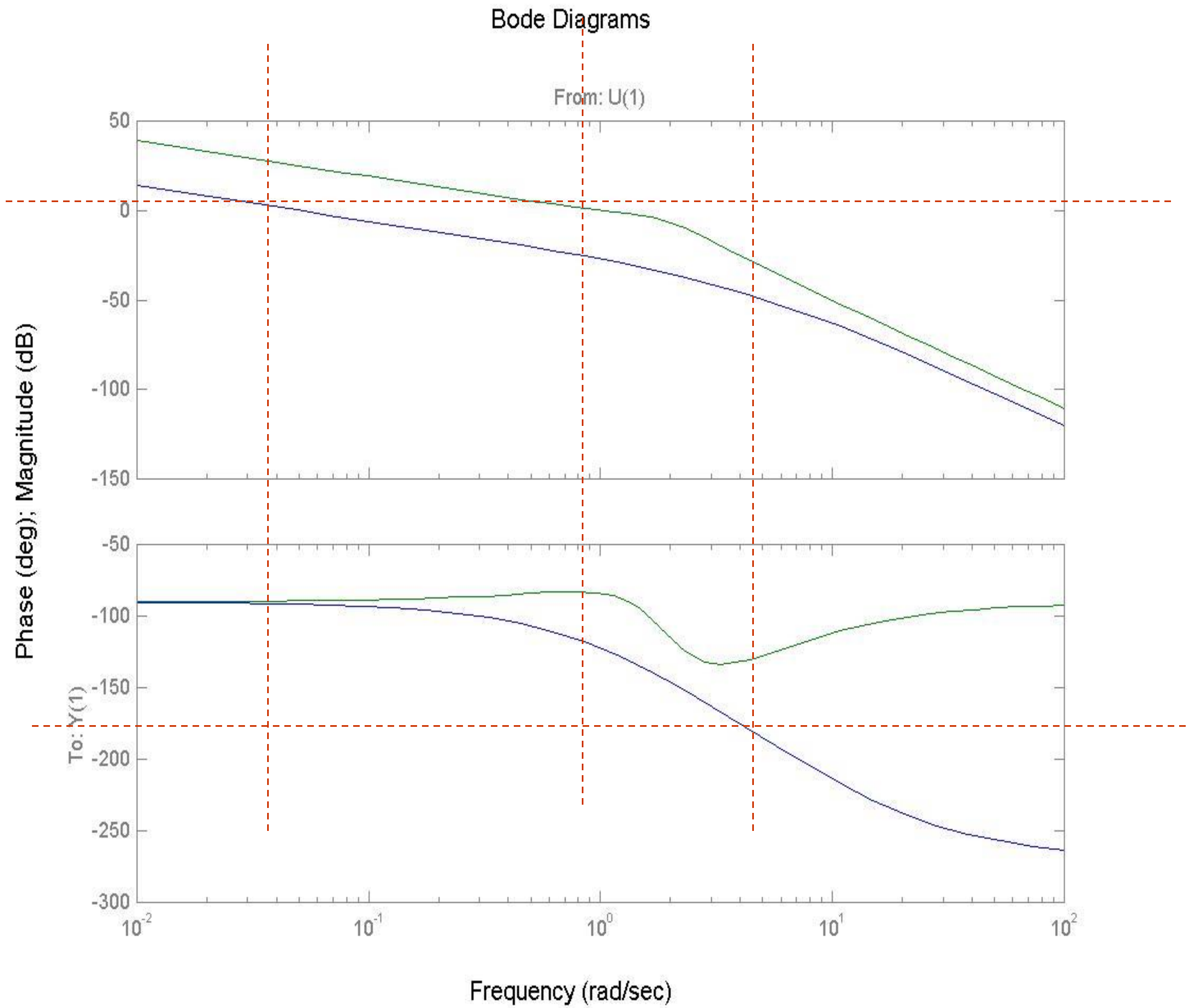
$$(-1,0) \Rightarrow \begin{cases} 0dB \\ -180^0 \end{cases}$$

Gain crossover frequency: ω_g

phase crossover frequency: ω_p



|GH|(dB)
G.M.<0
ω_g
ω
Unstable system
Stable system
∠GH
180^0
90^0
ω
−90^0
ω_p
−180^0
P.M.<0
Unstable system


Bode Diagrams
From: U(1)
Phase (deg); Magnitude (dB)
To: Y(1)
50
0
-50
-100
-150
-50
-100
-150
-200
-250
-300
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

Relative stability (Gain and Phase Margins)

- *A transfer function is called* ***minimum phase*** *when all the poles and zeros are LHP and* ***non-minimum-phase*** *when there are RHP poles or zeros.*

Minimum phase system $\Rightarrow$ Stable

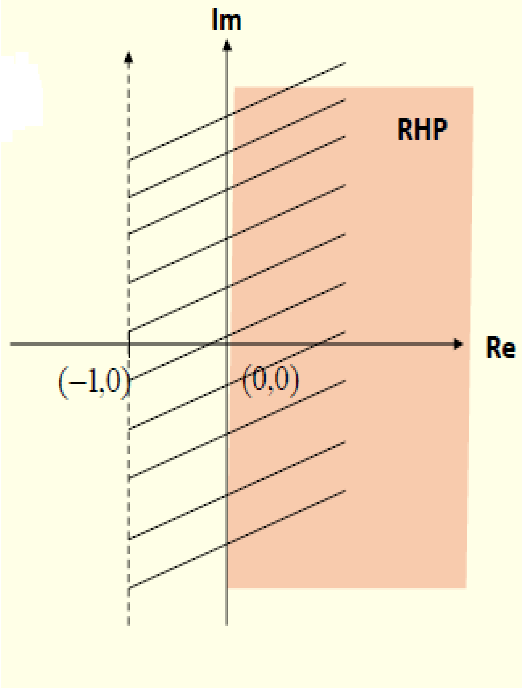
- ***The phase margin (PM):*** *is that amount of additional phase lag at the gain cross over frequency required to bring the system to the verge of instability.*

PM=180+[*phase of GH measured at the gain crossover frequency* (0 dB)]

The gain margin (GM): *is the reciprocal of the magnitude |G(jw)| at the frequency at which the phase angle is:*

GM=0 – [$dB\ of\ GH\ measured\ at\ the\ phase\ crossover\ frequency$ (– 180)]

- *Open loop transfer function:* $G(s)H(s)$
- *Closed-loop transfer function:* $1 + G(s)H(s)$
- *Open loop Stability → poles of* $G(s)\ H(s)$ in LHP
- *Closed-loop Stability → poles of* $G(s)\ H(s)$ in left side of (-1,0)



- The gain margin indicates the system gain can be increased by a factor of GM before the stability boundary is reached.

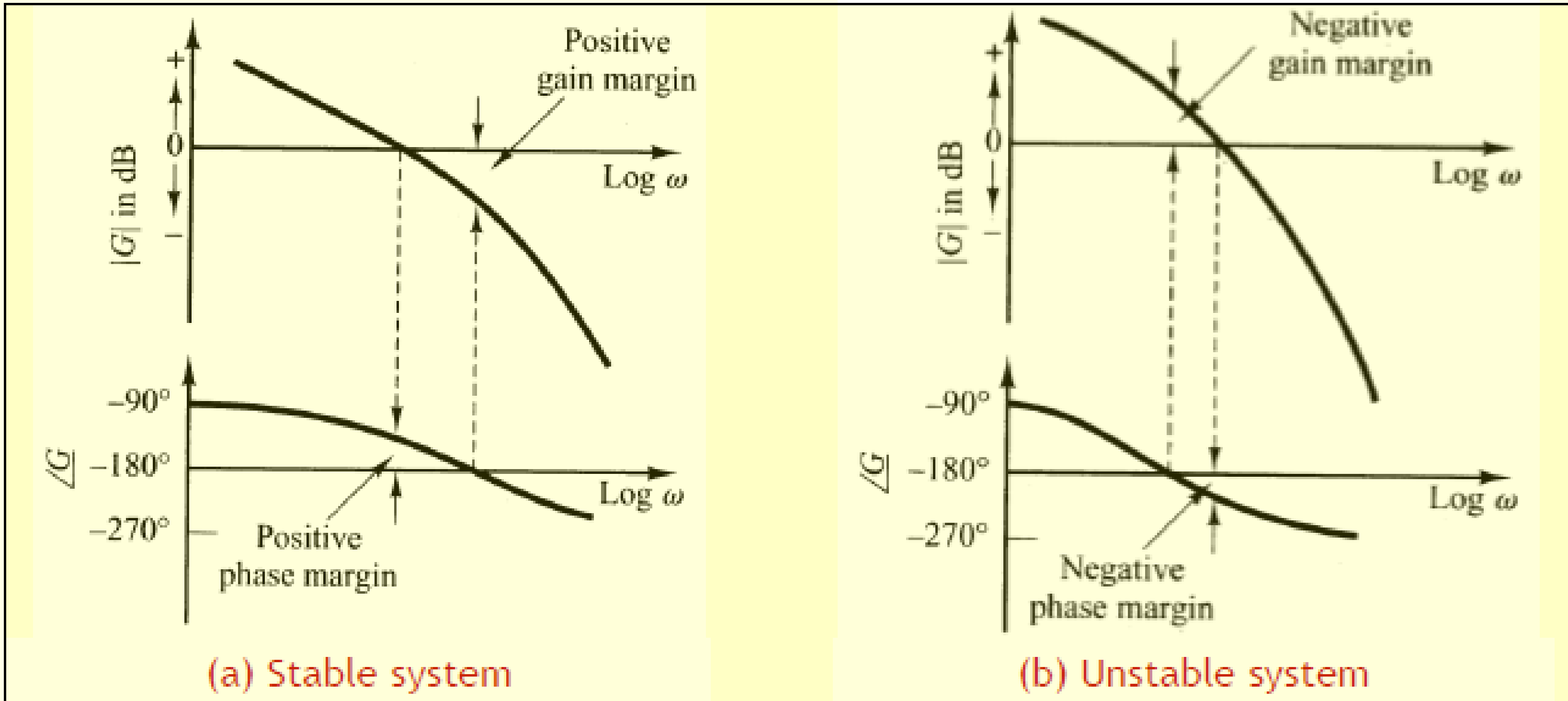
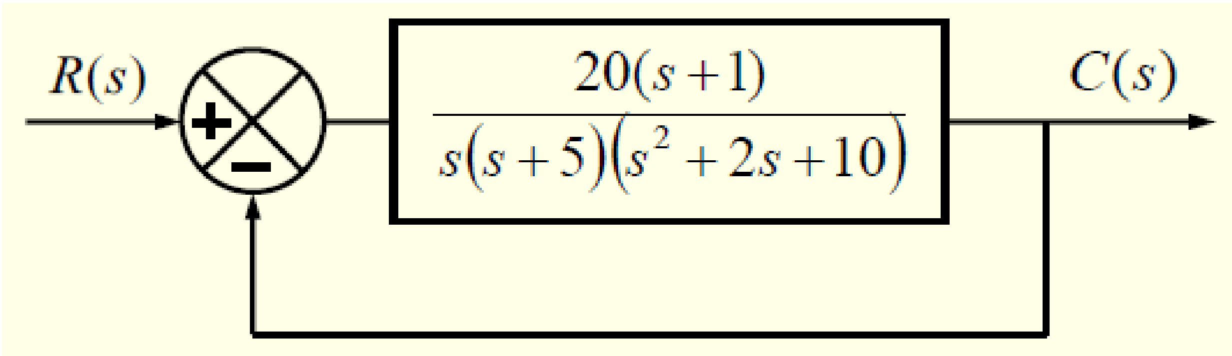


Fig. 5.17 Phase and gain margin definition

- The phase margin is the amount of phase shift of the system at unity magnitude that will result in stability.

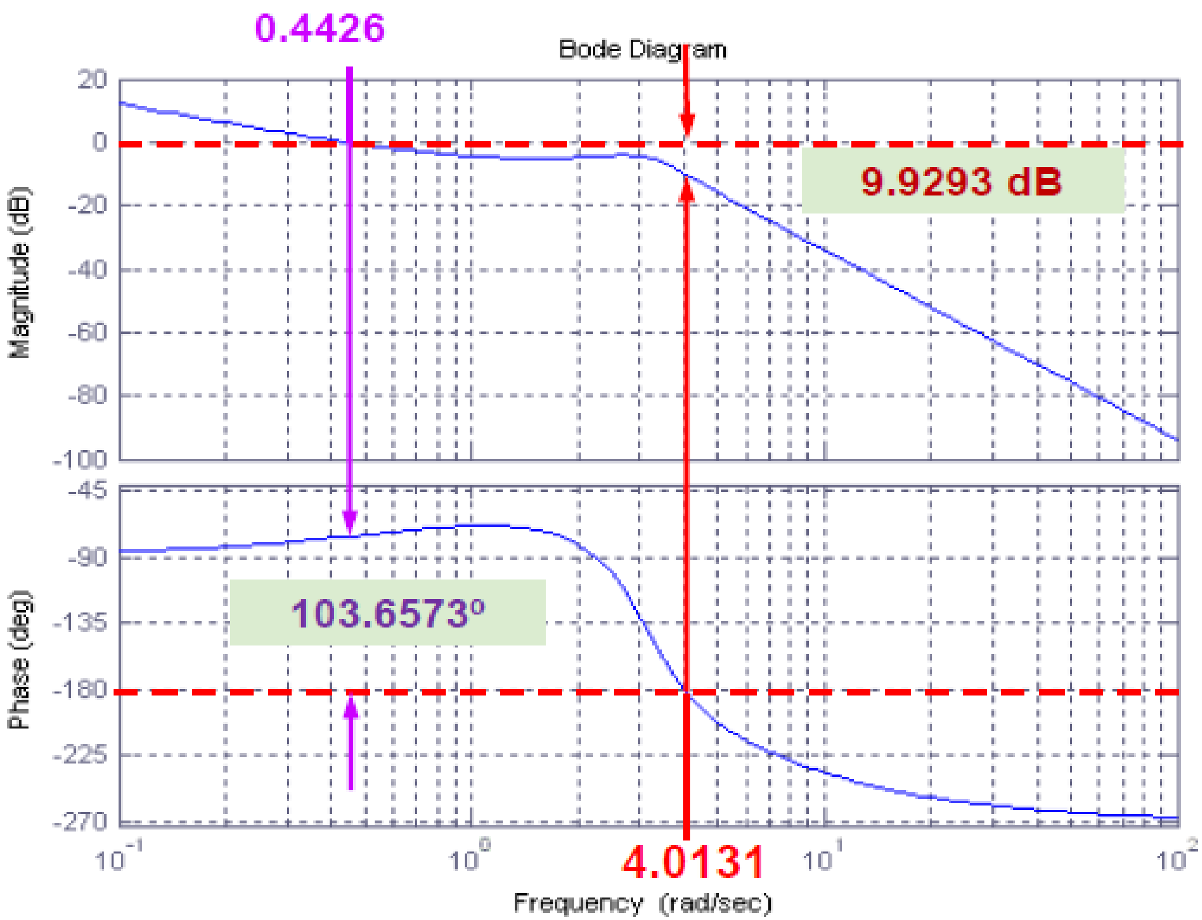
Example

- For the system shown, determine:
  - the gain margin.
  - phase margin.
  - phase-crossover frequency.
  - gain-crossover frequency.

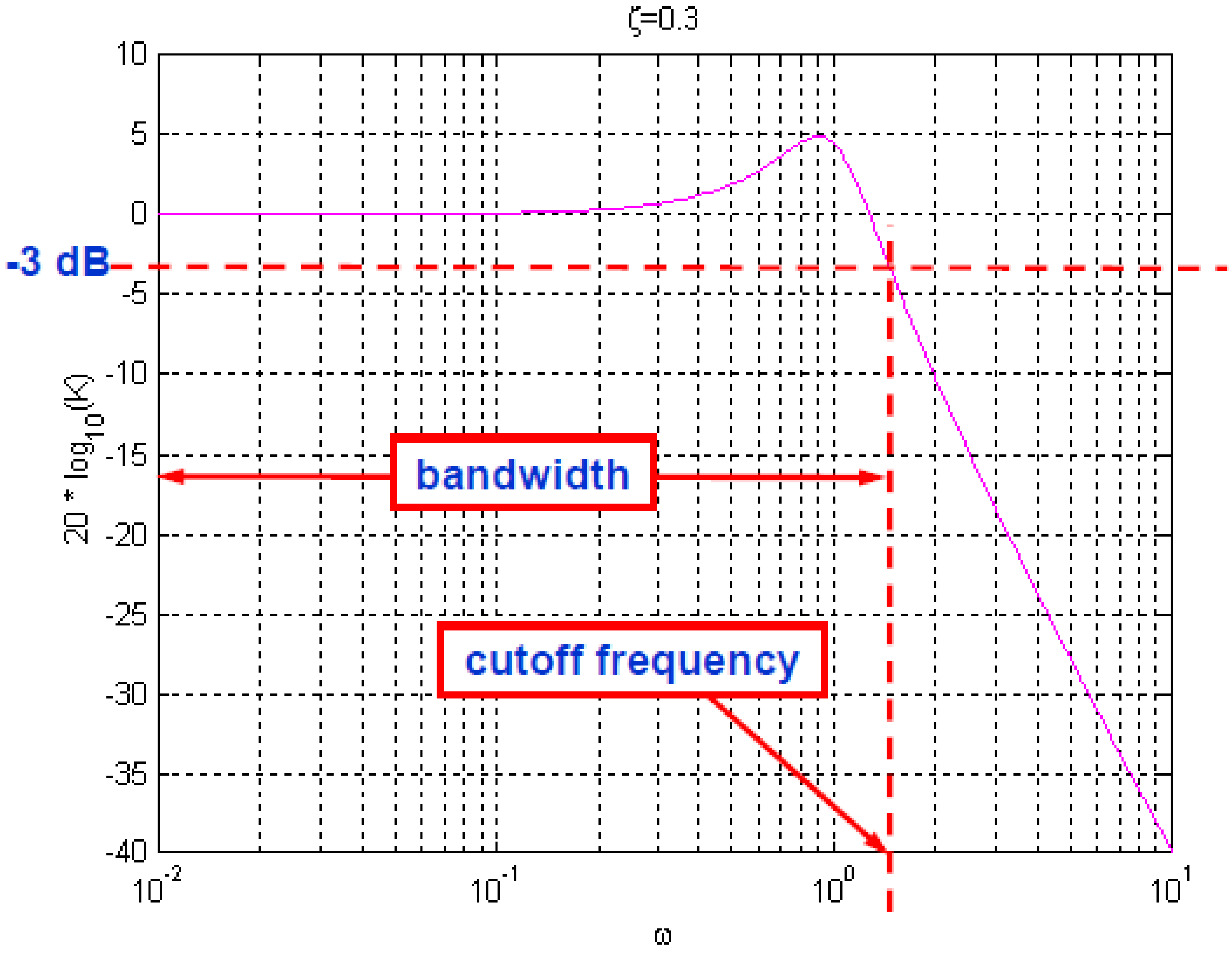


Solution

- The Bode plot for this system is shown in figure
- The ***gain margin= 9.929*** and the ***phase margin=103.7 degree.***



Bandwidth and Cutoff Frequency



Using Matlab For Frequency Response

- ***Instruction:*** We can use Matlab to run the frequency response for the previous example. We place the transfer function in the form:

$$\frac{5000(s+10)}{(s+1)(s+500)} = \frac{[5000s+50000]}{[s^2+501s+500]}$$

- ***The Matlab Program***

```
>> num= [5000 50000];
>> den = [1 501 500];
>> Bode (num,den)
```

Relationship between System Type & Log-Magnitude Curve

- Consider the unity-feedback control system. The static position, velocity, and acceleration error constants describe the low-frequency behavior of *type 0,type 1, and type 2* systems, respectively.

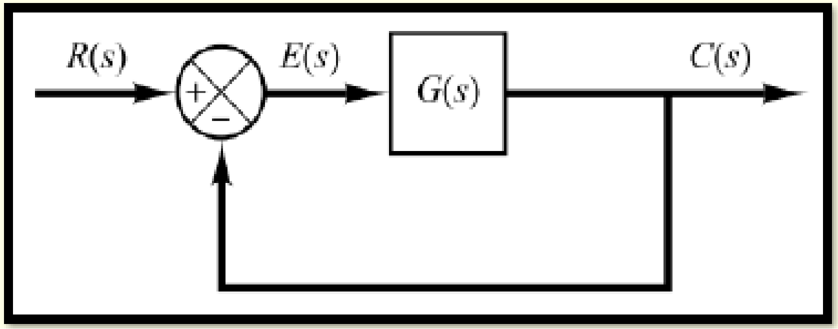
- For a given system, only one of the static error constants is finite and significant.

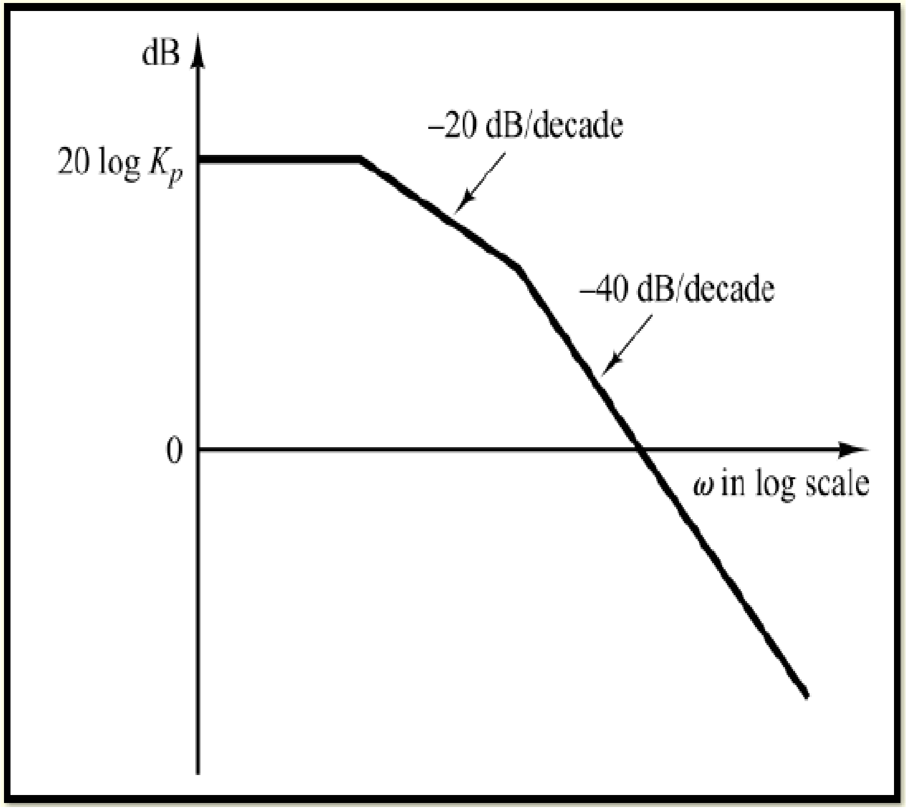
- The type of the system determines the slope of the log-magnitude curve at low frequencies.

- Thus, information concerning the existence and magnitude of the steady-state error of a control system to a given input can be determined from the observation of the low-frequency region of the log-magnitude curve.

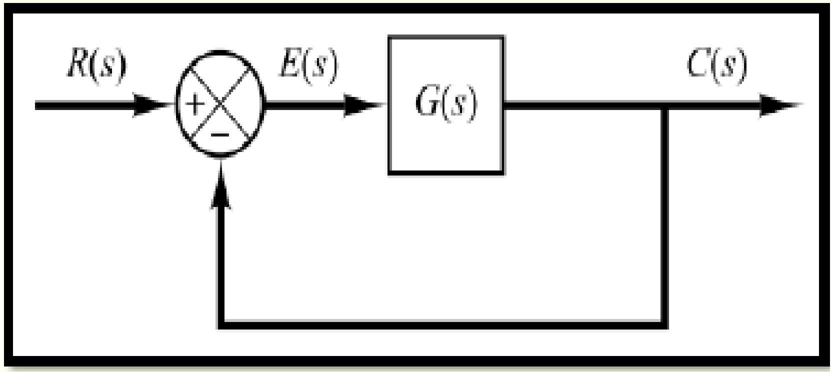
Determination of Static Position Error Constants.

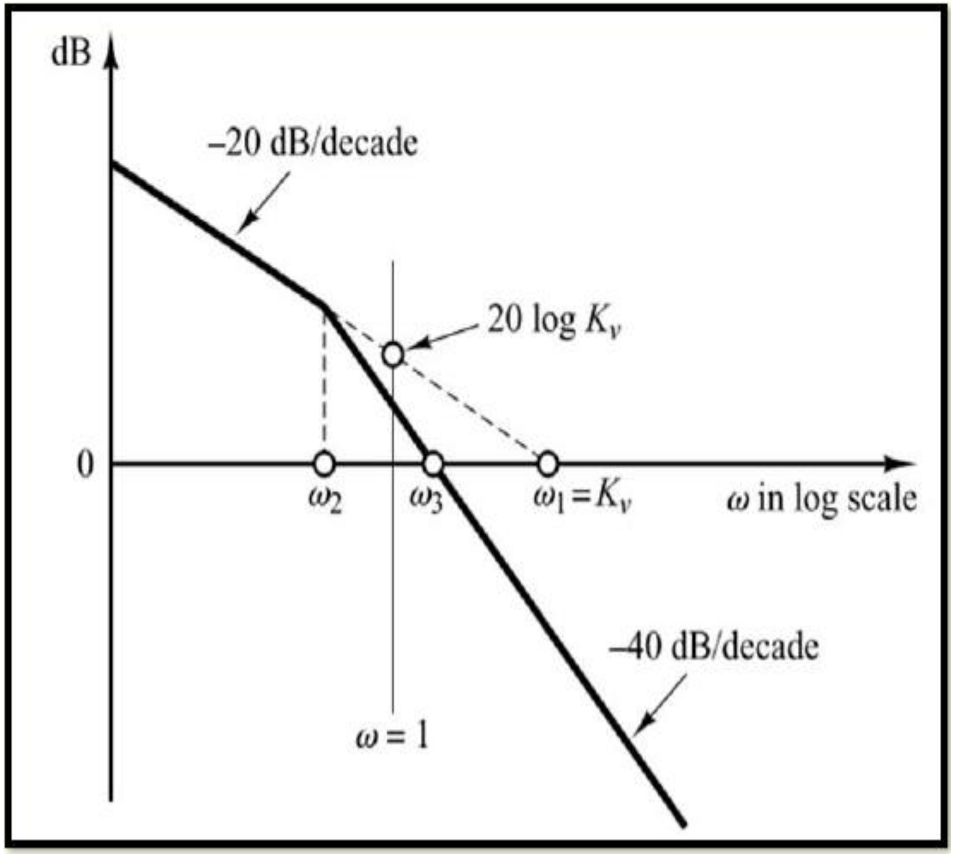
- ***Consider the type-0 unity feedback control system,***



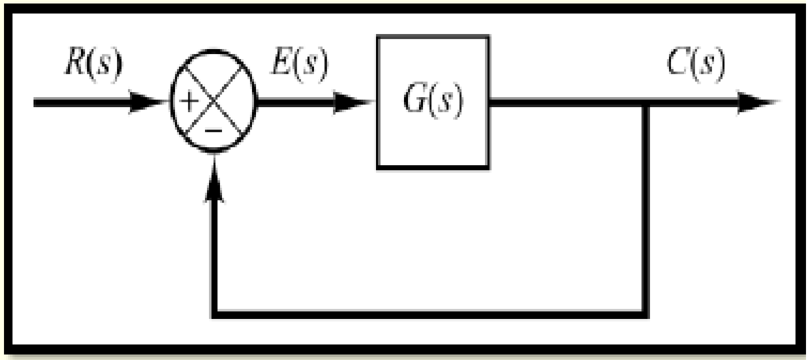


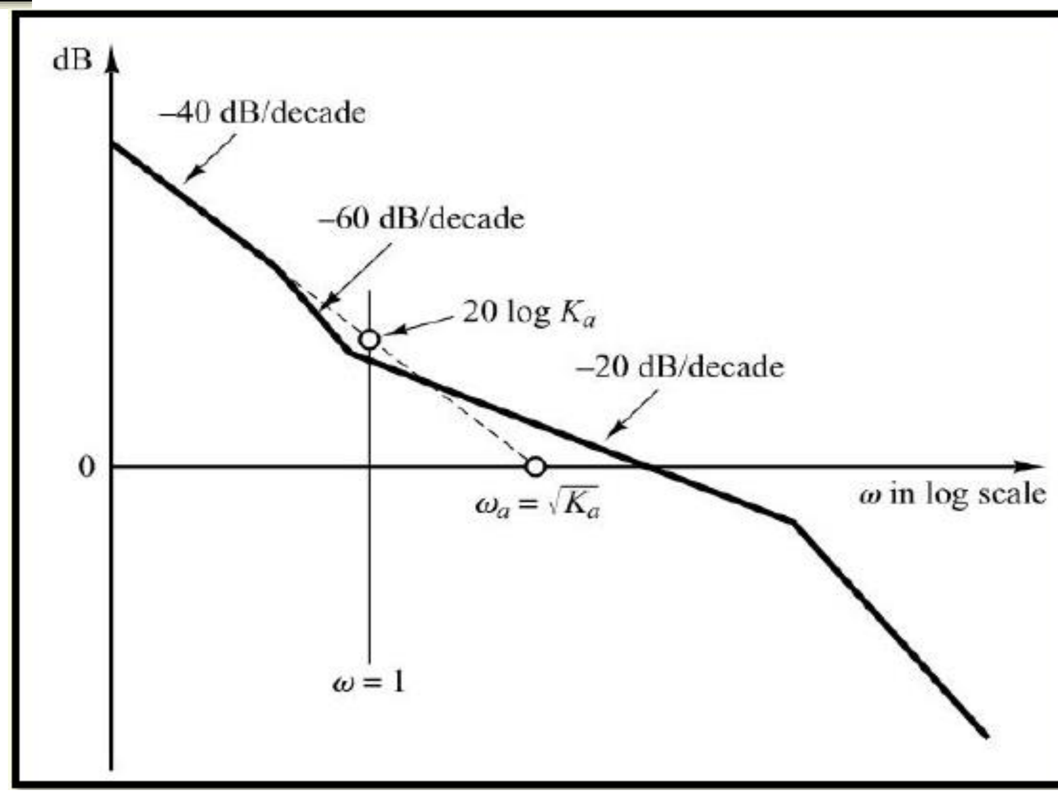
- ***Consider the type-1unity feedback control system,***





- ***Consider the type-2 unity feedback control system,***





Bode Plot Summary

- Straight-line approximations of the Bode plot may be drawn quickly from knowing the poles and zeroes
  - response approaches a minimum near the zeroes
  - response approaches a maximum near the poles
- The overall effect of constant, zero and pole terms

Term	Magnitude Break	Asymptotic Magnitude Slope	Asymptotic Phase Shift
Constant (K)	N/A	0	0°
Zero	upward	+20 dB/decade	+ 90°
Pole	downward	–20 dB/decade	– 90°

Bode Plot Summary

Express the transfer function in standard form

$$\mathbf{H}(j\omega)=\frac{K\left(j\omega\right)^{\pm N}(1+j\omega\tau_1)\left[1+2\varsigma_2(j\omega\tau_2)+(j\omega\tau_2)^2\right]\cdots}{(1+j\omega\tau_a)\left[1+2\varsigma_b(j\omega\tau_b)+(j\omega\tau_b)^2\right]\cdots}$$

There are four different factors:

- Constant gain term, *K*
- Poles or zeroes at the origin, $(j\omega)^{\pm N}$
- Poles or zeroes of the form $(1+ j\omega\tau)$
- Quadratic poles or zeroes of the form $1+2\zeta(j\omega\tau)+(j\omega\tau)^2$

Bode Plot Summary

We can combine the constant gain term (*K*) and the *N* pole(s) or zero(s) at the origin such that the magnitude crosses 0 dB at

$$Pole: \quad \frac{K}{(j\omega)^N} \qquad \omega_{0\,dB} = K^{1/N}$$

$$Zero: \quad K(j\omega)^N \qquad \omega_{0\,dB} = (1/K)^{1/N}$$

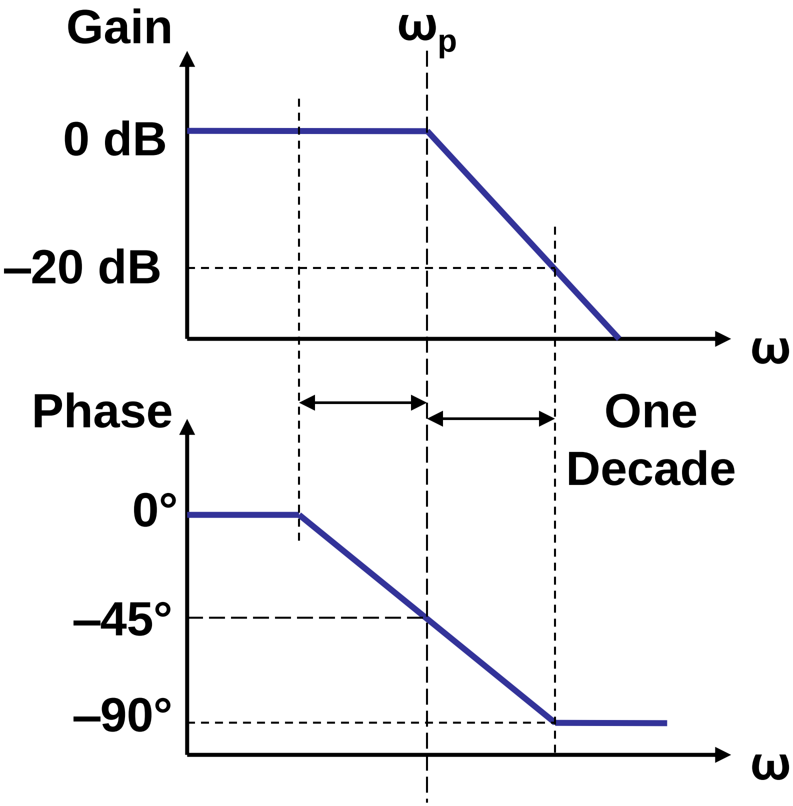
Define the *break frequency* to be at $\omega=1/\tau$ with magnitude at ±3 dB and phase at ±45°

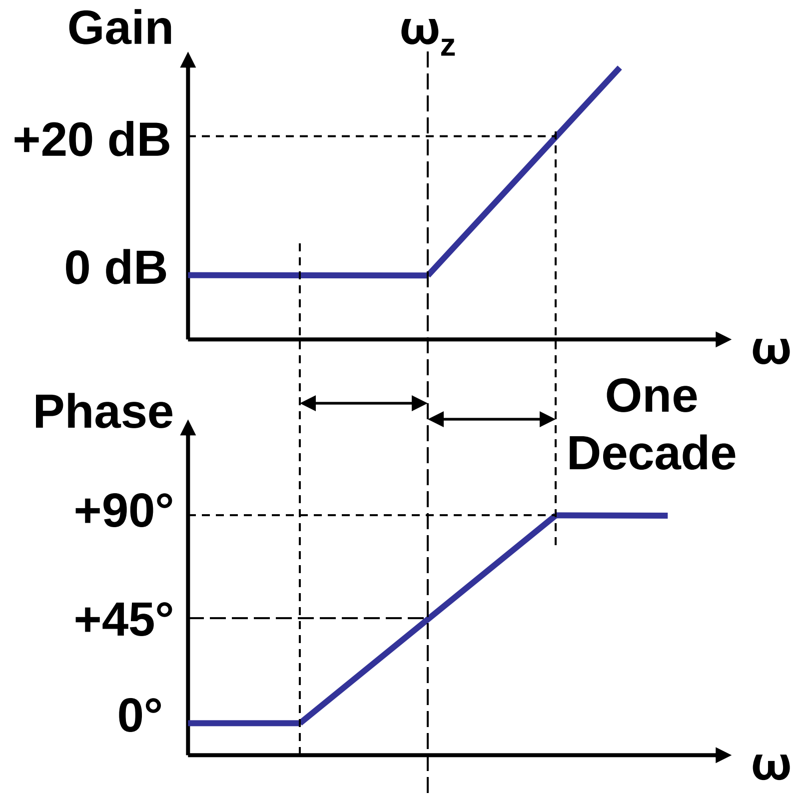
Bode Plot Summary

Factor	Magnitude Behavior			Phase Behavior		
	Low Freq	Break	Asymptotic	Low Freq	Break	Asymptotic
Constant	$20 \log_{10}(K)$ for all frequencies			0° for all frequencies		
Poles or zeros at origin	±20N dB/decade for all frequencies with a crossover of 0 dB at $\omega=1$			±90°(N) for all frequencies		
First order (simple) poles or zeros	0 dB	±3N dB at $\omega=1/\tau$	±20N dB/decade	0°	±45°(N) with slope ±45°(N) per decade	±90°(N)
Quadratic poles or zeros	0 dB	see ζ at $\omega=1/\tau$	±40N dB/decade	0°	±90°(N)	±180°(N)

where N is the number of roots of value τ

Single Pole & Zero Bode Plots





Pole at
$\omega_p=1/\tau$

Assume K=1
$20 \log_{10}(K) = 0$ dB

Zero at
$\omega_z=1/\tau$

Bode Plot Refinements

Further refinement of the magnitude characteristic for first order poles and zeros is possible since

- Magnitude at half break frequency: $|\mathbf{H}(\frac{1}{2}\omega_b)| = \pm 1$ dB
- Magnitude at break frequency: $|\mathbf{H}(\omega_b)| = \pm 3$ dB
- Magnitude at twice break frequency: $|\mathbf{H}(2\omega_b)| = \pm 7$ dB

Second order poles (and zeros) require that the *damping ratio* (ζ value) be taken into account; see Fig. 9-30 in textbook

Bode Plots to Transfer Function

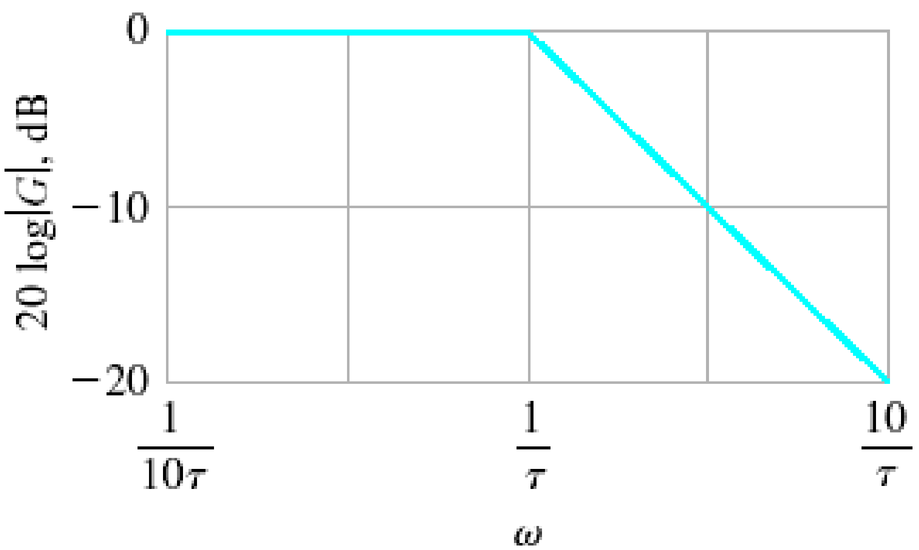
We can also take the Bode plot and extract the transfer function from it (although in reality there will be error associated with our extracting information from the graph)

First, determine the constant gain factor, K

Next, move from lowest to highest frequency noting the appearance and order of the poles and zeros

Frequency Response Plots

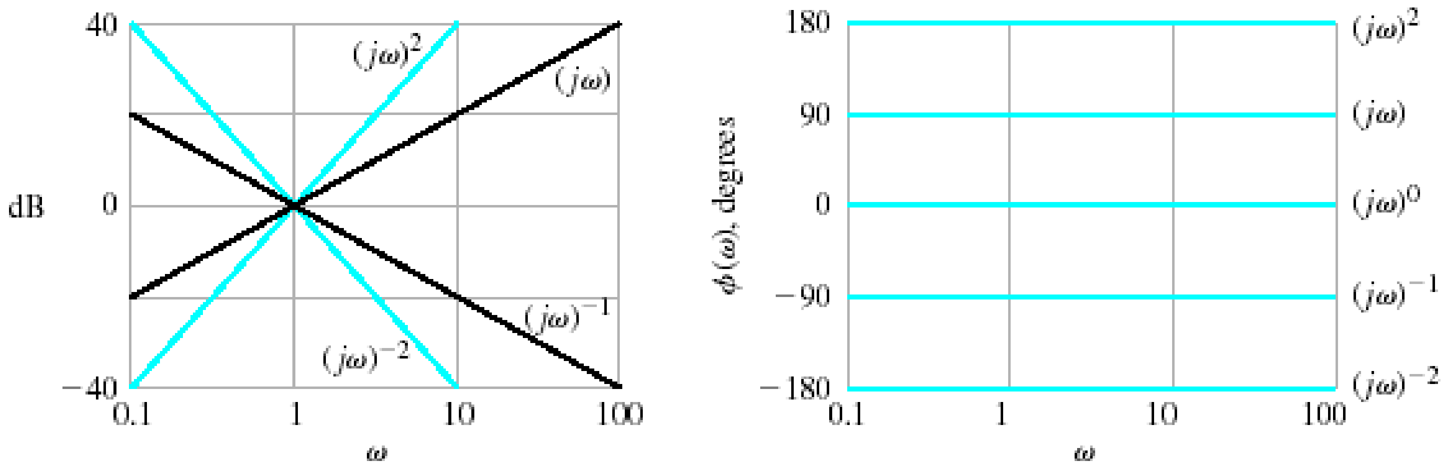
Bode Plots – Real Poles (Graphical Construction)



Asymptotic curve for $(j\omega\tau + 1)^{-1}$.

Frequency Response Plots

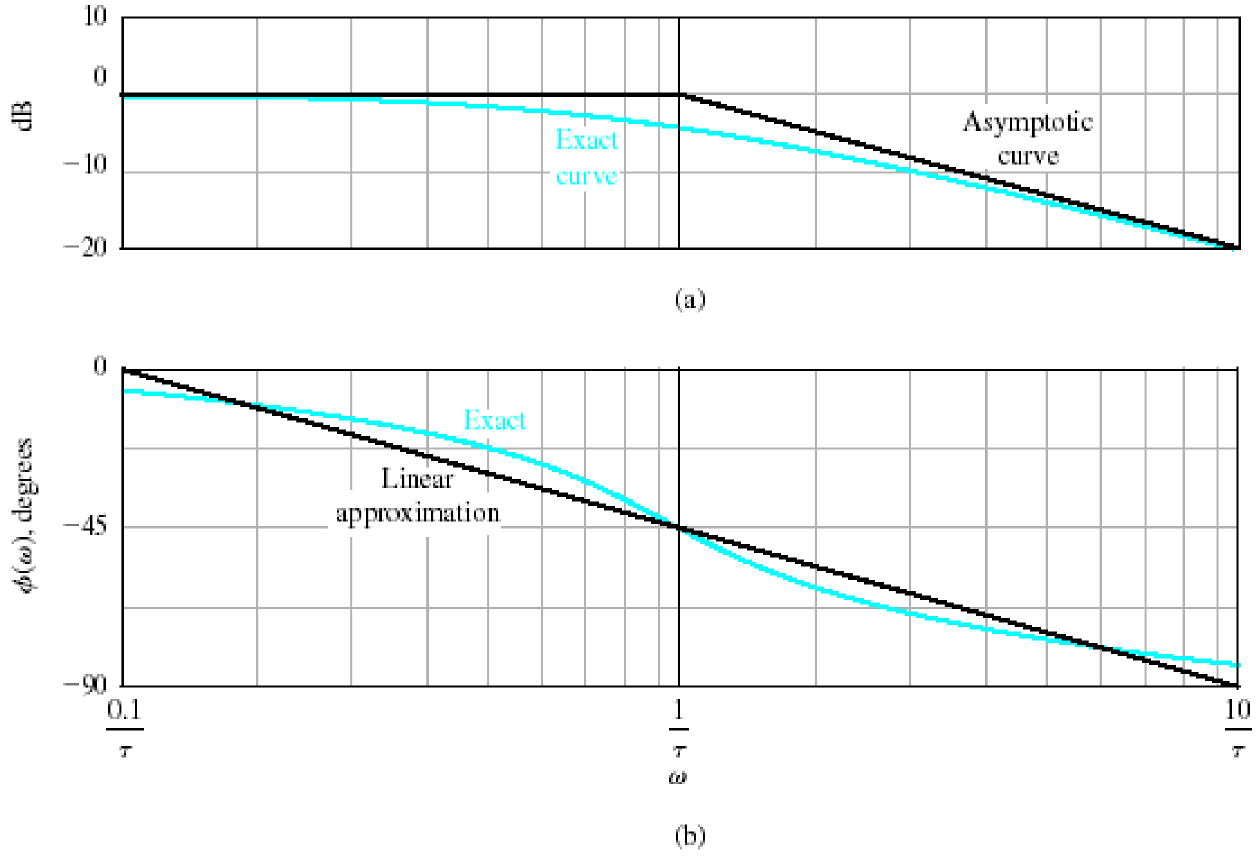
Bode Plots – Real Poles



Bode diagram for $(j\omega)^{\pm N}$.

Frequency Response Plots

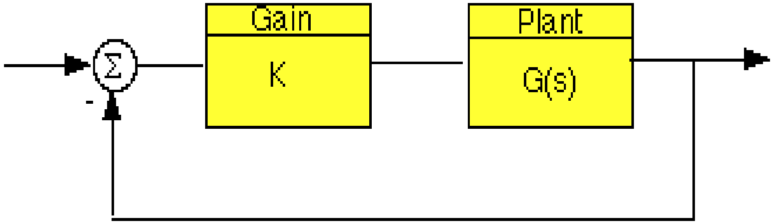
Bode Plots – Real Poles



Bode diagram for $(1 + j\omega\tau)^{-1}$.

Gain and Phase Margin

Let's say that we have the following syste



where K is a variable (constant) gain and G(s) is the plant under consideration.

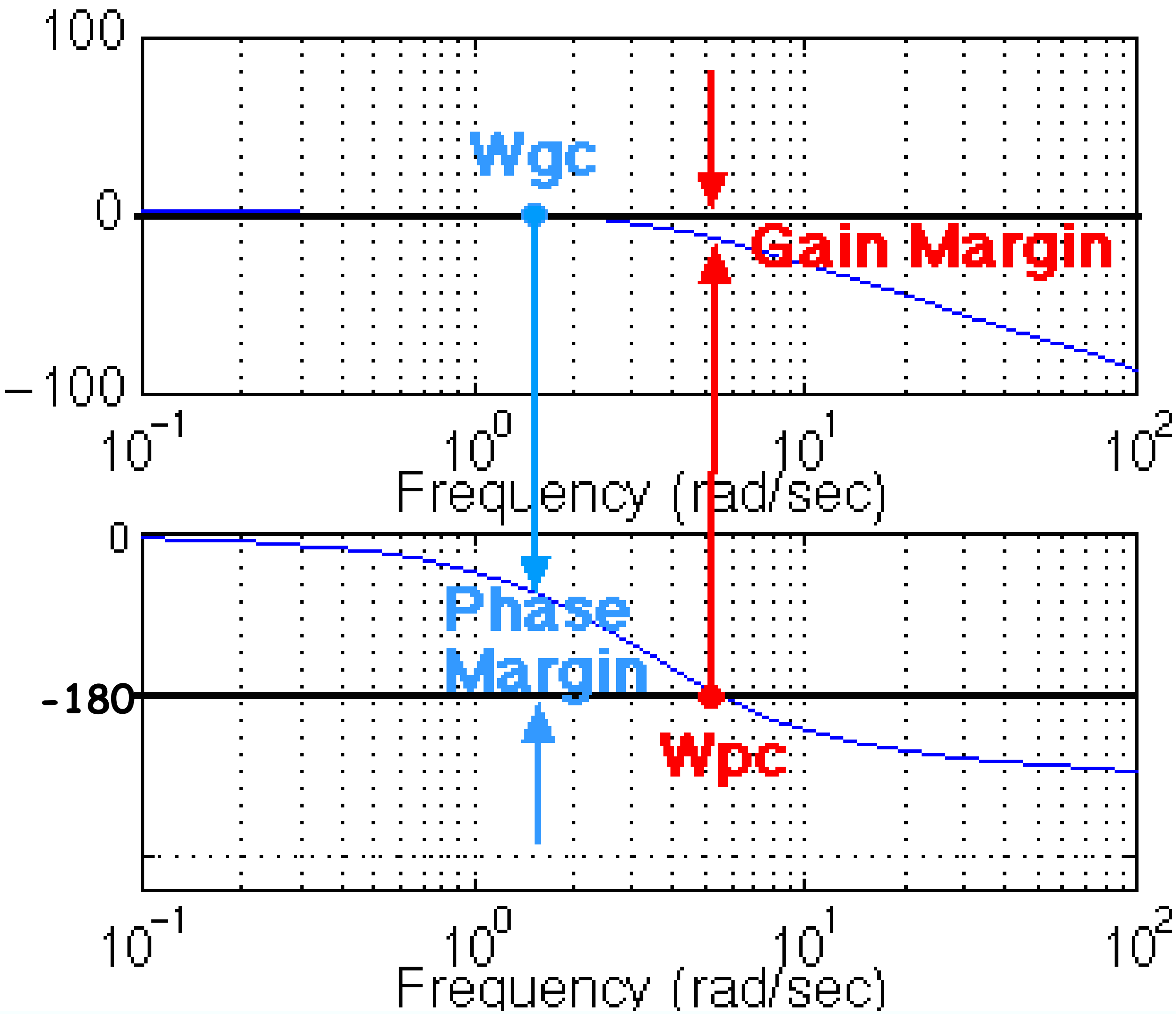
The gain margin is defined as the change in open loop gain required to make the system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed loop. Keep in mind that unity gain in magnitude is equal to a gain of zero in dB

The phase margin is defined as the change in open loop phase shift required to make a closed loop system unstable.

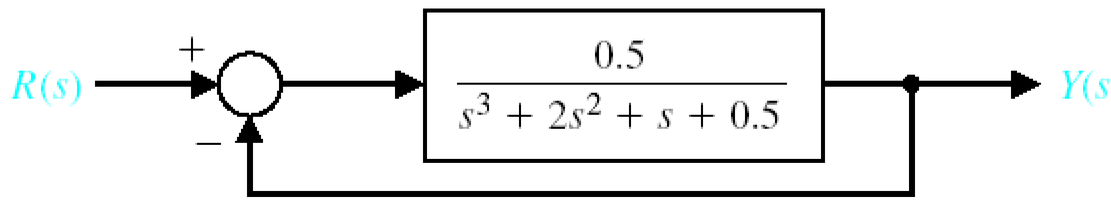
The phase margin is the difference in phase between the phase curve and -180 deg at the point corresponding to the frequency that gives us a gain of 0dB (the gain cross over frequency, Wgc).

Likewise, the gain margin is the difference between the magnitude curve and 0dB at the point corresponding to the frequency that gives us a phase of -180 deg (the phase cross over frequency, Wpc).

Gain and Phase Margin



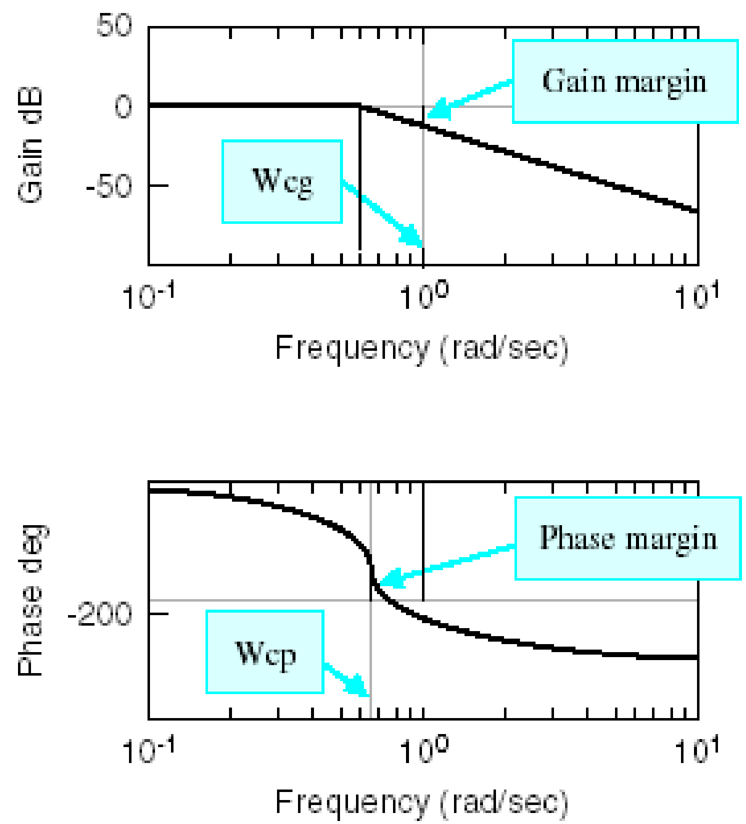
Examples - Bode




```
[mag,phase,w]=bode(sys);
[Gm,Pm,Wcg,Wcp]=margin(mag,phase,w);
```

or

```
[Gm,Pm,Wcg,Wcp]=margin(sys);
```

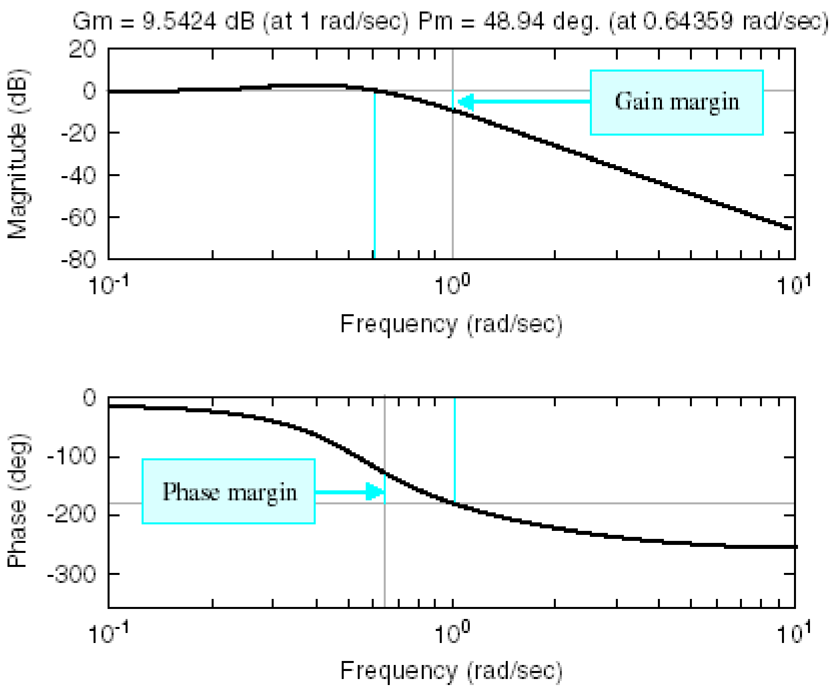


Example

```
num=[0.5]; den=[1 2 1 0.5];
sys=tf(num,den);
margin(sys);
```

Gm = gain margin (dB)
Pm = phase margin (deg)
Wcg = freq. for phase = −180
Wcp = freq. for gain = 0 dB

Examples - Bode

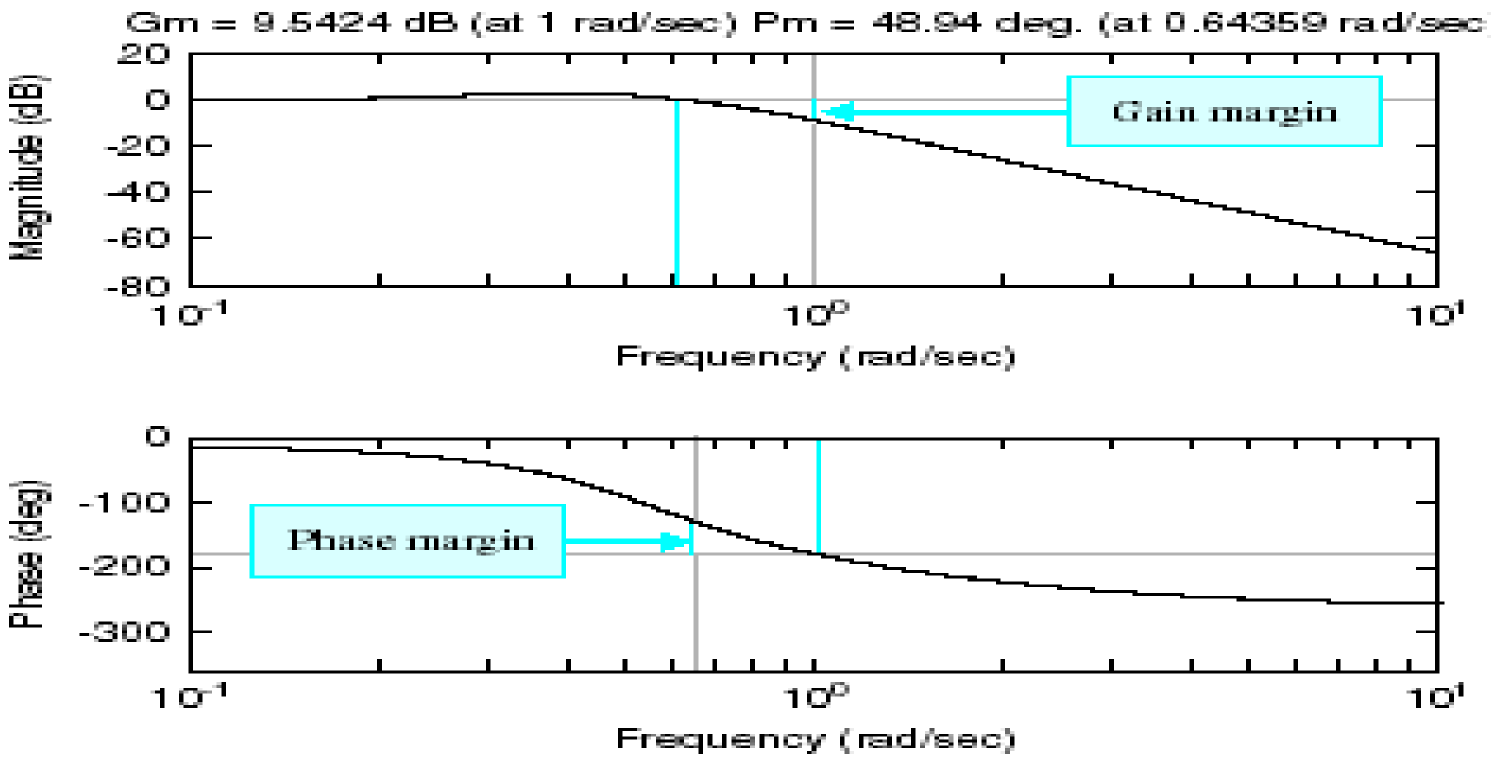



```
num=[0.5];
den=[1 2 1 0.5 ];
sys=tf(num,den);
%
w=logspace(-1,1,200);
%
[mag,phase,w]=bode(sys,w);
%
margin(mag,phase,w);
```

Open-loop system

Specify frequency range

Examples – Bode

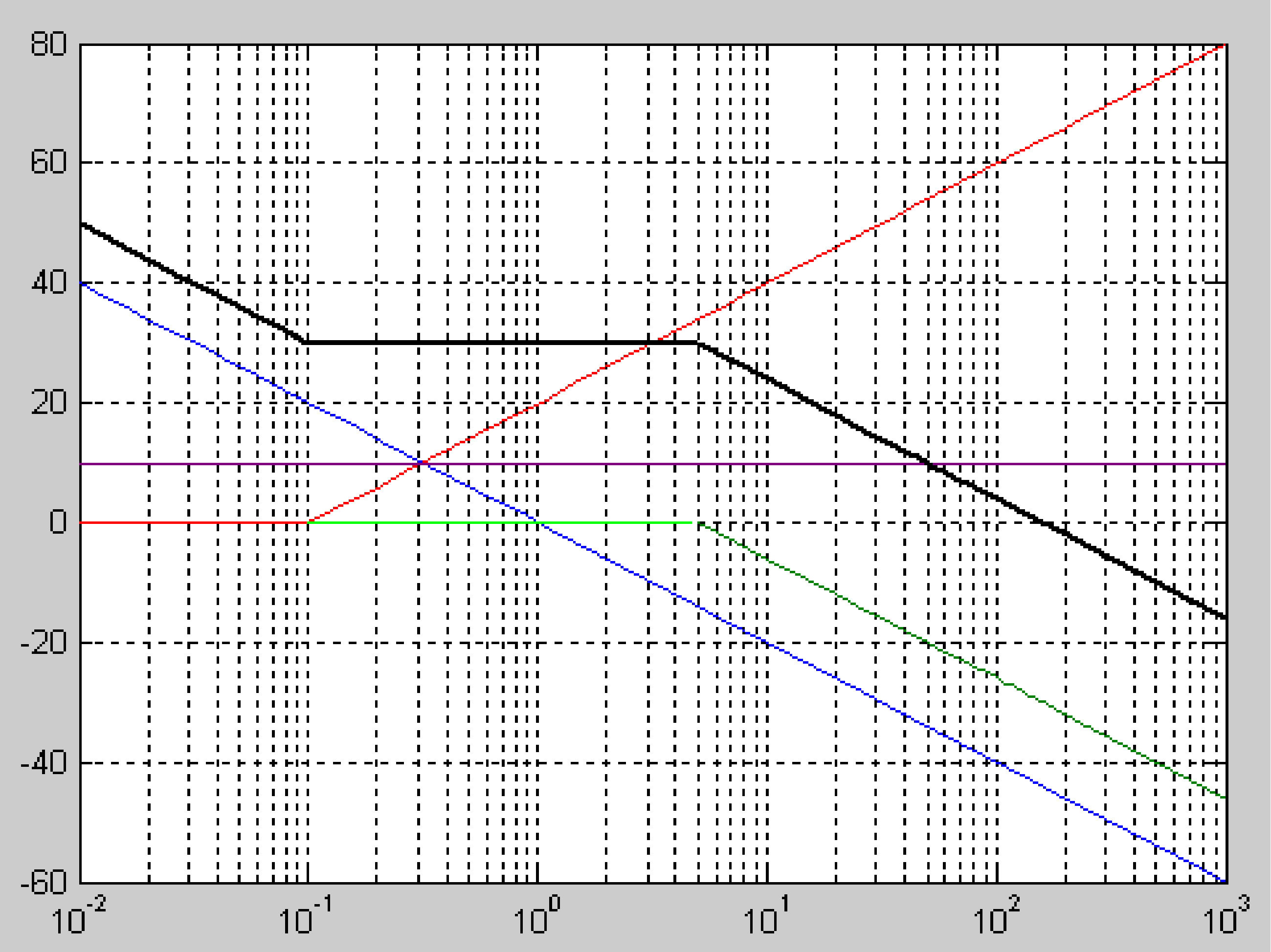



```
num=[0.5];
den=[1 2 1 0.5 ];
sys=tf(num,den);
%
w=logspace(-1,1,200);
%
[mag,phase,w]=bode(sys,w);
%
margin(mag,phase,w);
```

Open-loop system

Specify frequency range

Magnitude Bode plot of $\dfrac{\sqrt{10}(1+{}^{j\omega}/_{0.1})}{j\omega(1+{}^{j\omega}/_{5})}$



-- $20\log_{10}(1+j\omega/0.1)$

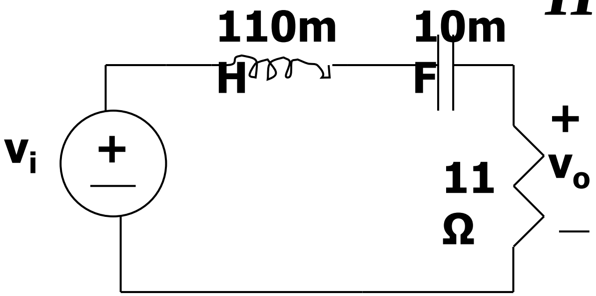
-- $-20\log_{10}(1+j\omega/5)$

-- $-20\log_{10}(\omega)$

-- $20\log_{10}(\sqrt{10})$

-- $20\log_{10}|H(j\omega)|$

EXAMPLE



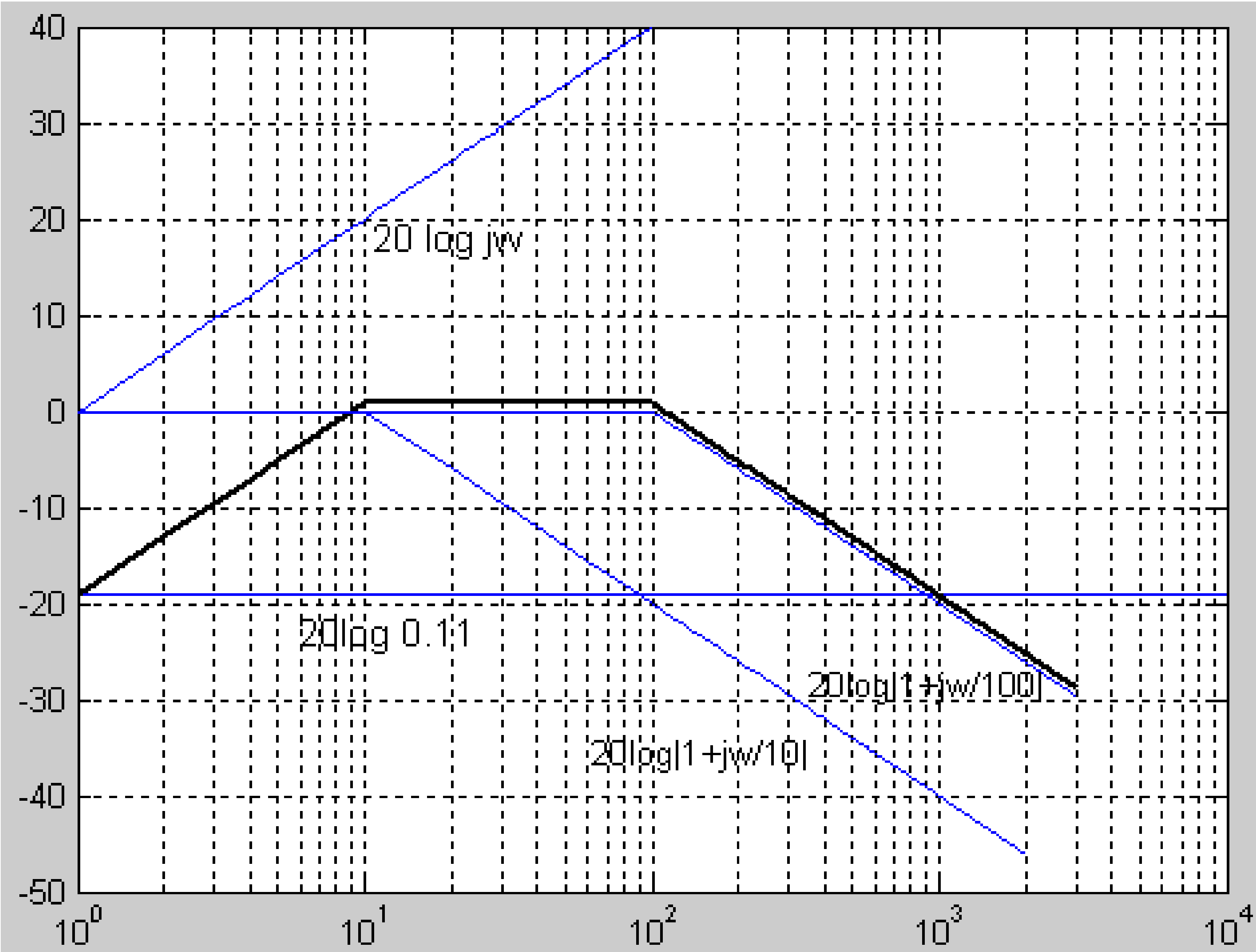
$$H(s)=\frac{(R/L)s}{s^2+(R/L)s+\dfrac{1}{LC}}=\frac{110s}{s^2+110s+1000}$$

$$=\frac{110s}{(s+10)(s+100)}$$

$$H(j\omega)=\frac{0.11j\omega}{\left[1+j\left(\omega/10\right)\right]\left[1+j\left(\omega/100\right)\right]}$$

$$A_{dB}=20\log_{10}\left|H(j\omega)\right|$$

$$=20\log_{10}0.11+20\log_{10}\left|j\omega\right|-20\log_{10}\left|1+j\frac{\omega}{10}\right|-20\log_{10}\left|1+j\frac{\omega}{100}\right|$$



40
30
20
10
0
-10
-20
-30
-40
-50
10^0
10^1
10^2
10^3
10^4
20 log jw
20log 0.11
20log|1+jw/100|
20log|1+jw/10|

Calculate 20log$_{10}$ |H(jω)| at ω=50 rad/s and ω=1000 rad/s

$$H(j50) = \frac{0.11(j50)}{(1+j5)(1+j0.5)} = 0.9648 \angle\text{-}15.25^0$$

$$20\log_{10}\left|H(j50)\right| = 20\log_{10}(0.9648) = -0.311\ \text{dB}$$

$$H(j1000) = \frac{0.11(j1000)}{(1+j100)(1+j10)} = 0.1094\angle -83.72^0$$

$$20\log_{10}\left|H(j1000)\right| = 20\log_{10}(0.1094) = -19.22\ \text{dB}$$

Using the Bode diagram, calculate the amplitude of v_o if $v_i(t)=5\cos(500t+15^0)$V.

From the Bode diagram, the value of A_{dB} at ω=500 rad/s is approximately -12.5 dB. Therefore,

$$|A| = 10^{(-12.5/20)} = 0.24$$

$$V_{mo} = |A| V_{mi} = (0.24)(5) = 1.2V$$

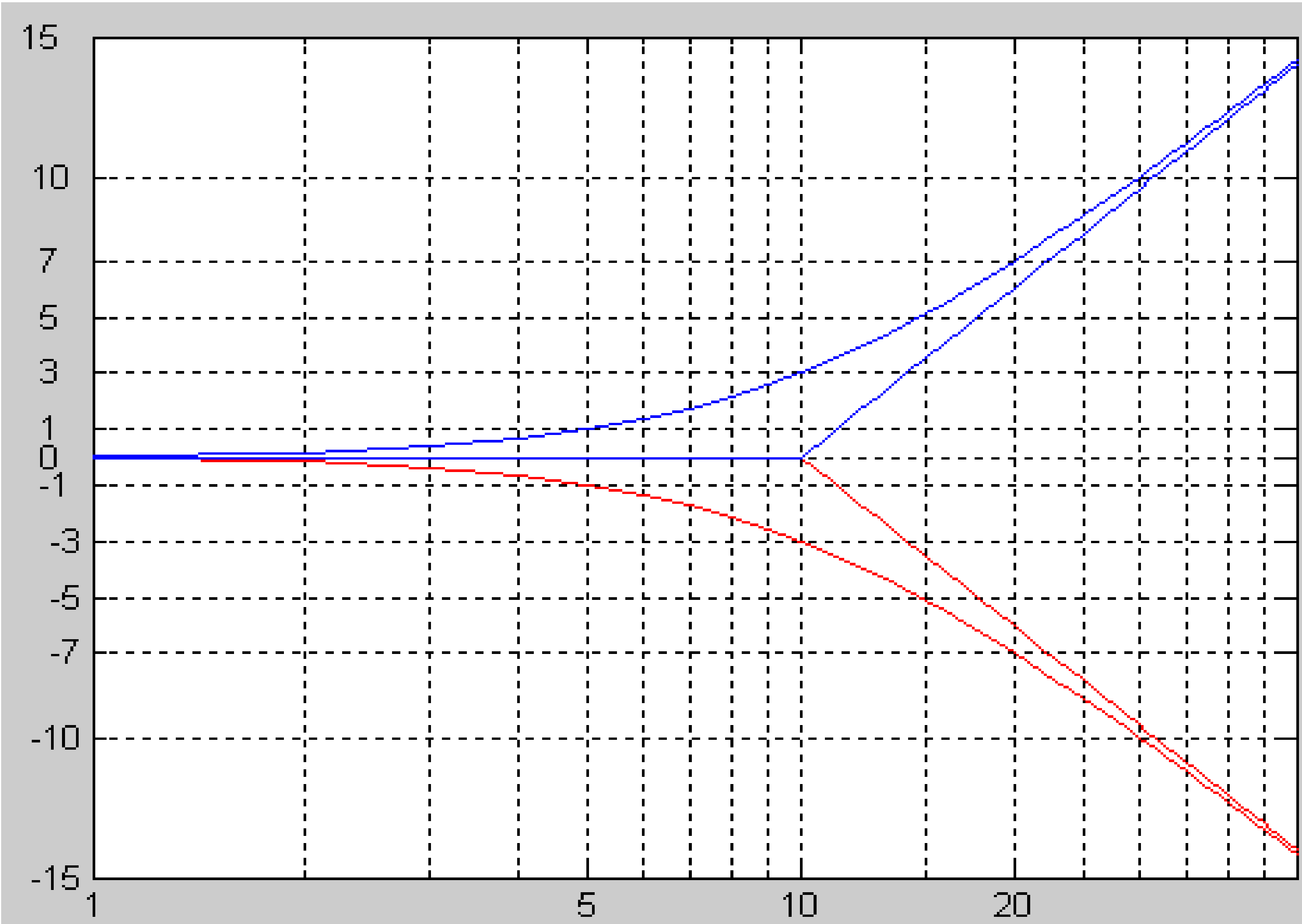
The straight-line plots for first-order poles and zeros can be made more accurate by correcting the amplitude values at the corner frequency, one half the corner frequency, and twice the corner frequency. The actual decibel values at these frequencies

$$A_{dB_c} = \pm 20\log_{10}\left|1 + j\,1\right| = \pm 20\log_{10}\sqrt{2} \approx \pm 3\ \text{dB}$$

$$A_{dB_{c/2}} = \pm 20\log_{10}\left|1 + j\,1/2\right| = \pm 20\log_{10}\sqrt{5/4} \approx \pm 1\ \text{dB}$$

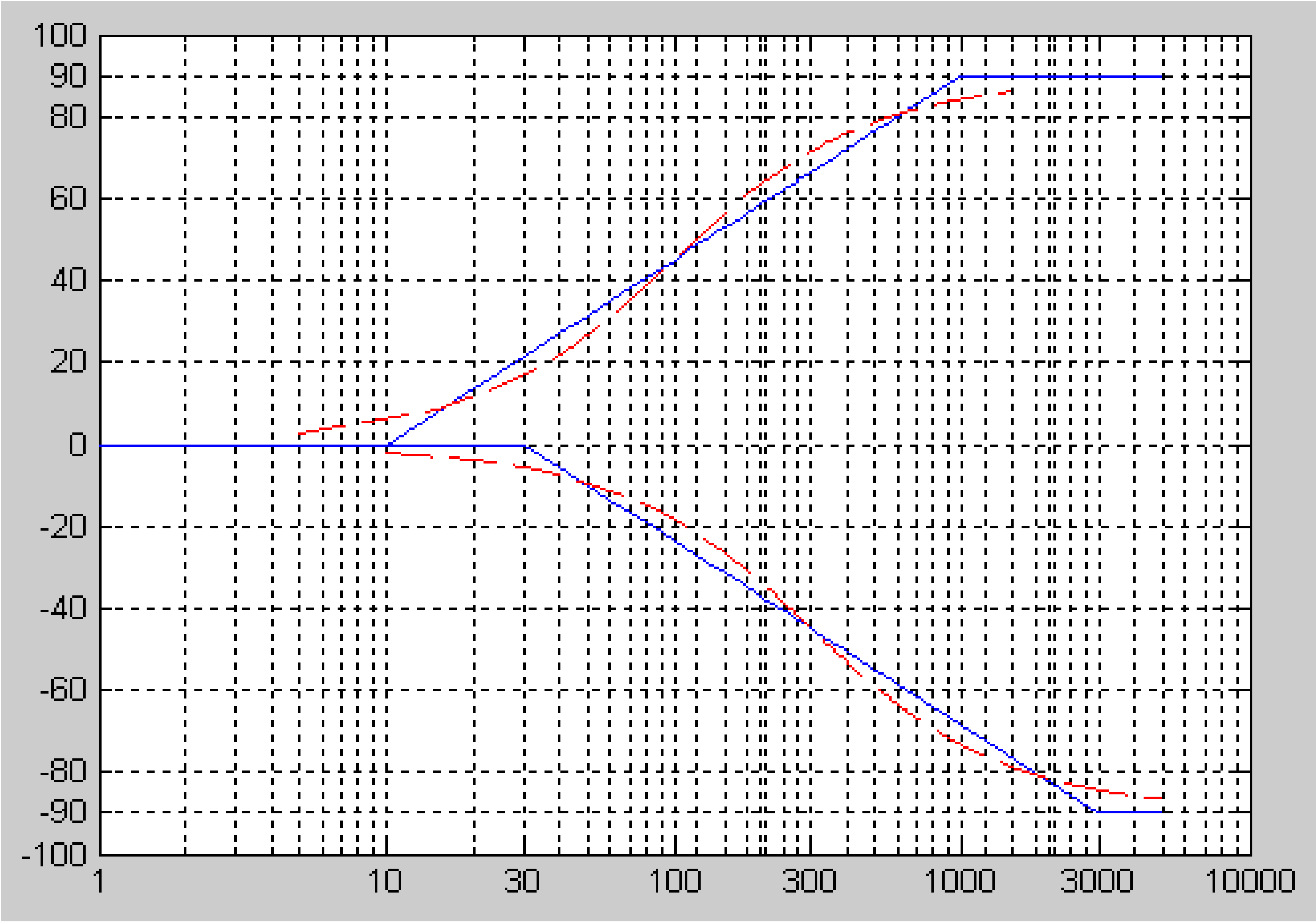
$$A_{dB_{2c}} = \pm 20\log_{10}\left|1 + j\,2\right| = \pm 20\log_{10}\sqrt{5} \approx \pm 7\text{dB}$$

In these equations, + sign corresponds to a first-order zero, and – sign is for a first-order pole.



15
10
7
5
3
1
0
-1
-3
-5
-7
-10
-15
1
5
10
20

1. **The phase angle for constant K_o is zero.**
2. **The phase angle for a first-order zero or pole at the origin is a constant $\pm 90^0$.**
3. **For a first-order zero or pole not at the origin,**
   - **For frequencies less than one tenth the corner frequency, the phase angle is assumed to be zero.**
   - **For frequencies greater than 10 times the corner frequency, the phase angle is assumed to be $\pm 90^0$.**
   - **Between these frequencies the plot is a straight line that goes from 0^0 to $\pm 90^0$ with a slope of $\pm 45^0$/decade.**


100
90
80
60
40
20
0
-20
-40
-60
-80
-90
-100
1
10
30
100
300
1000
3000
10000

$$H(j\omega) = \frac{0.11(j\omega)}{[1 + j(\omega/10)][1 + j(\omega/100)]}$$

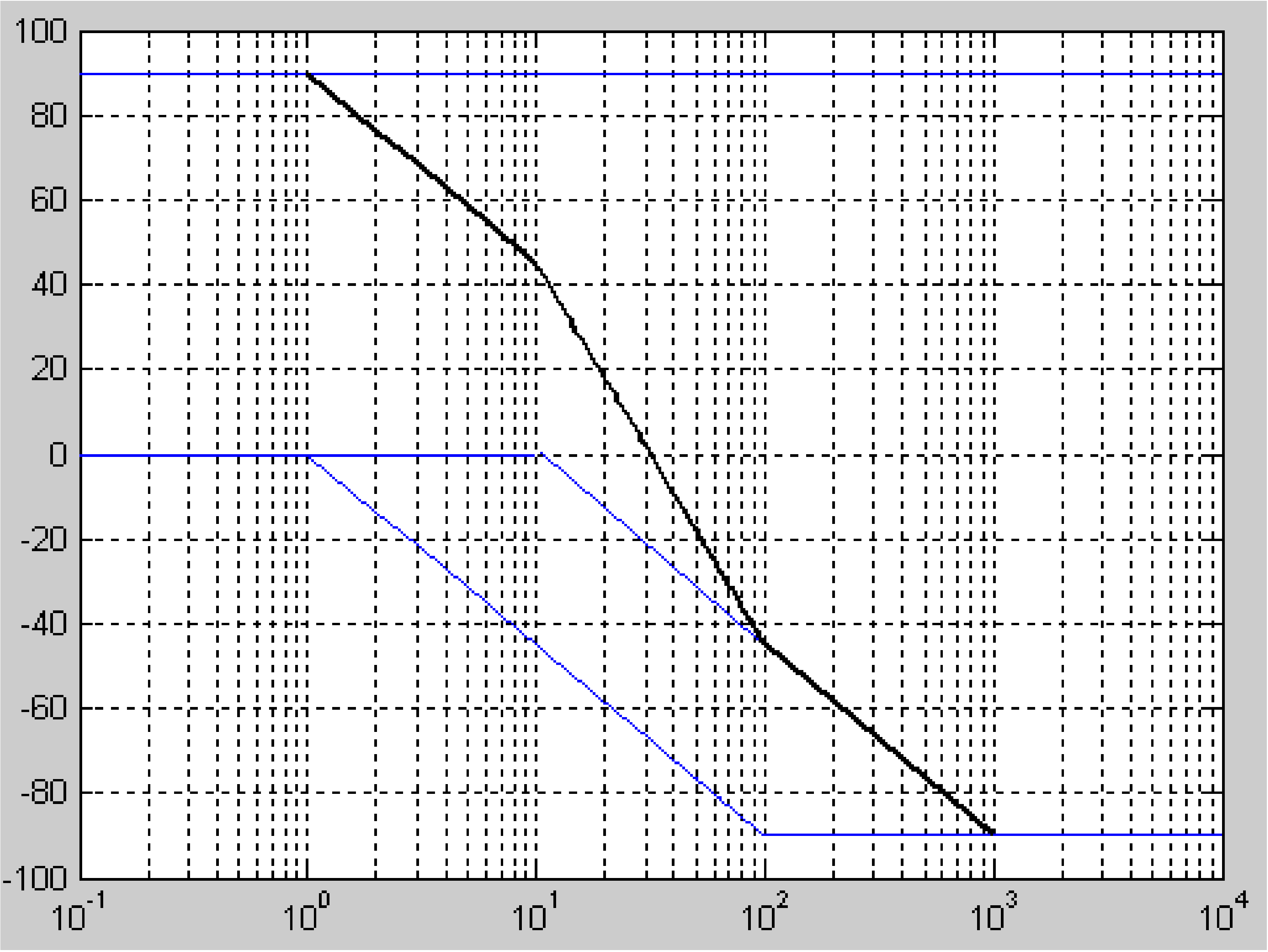
$$= \frac{0.11|j\omega|}{|1 + j(\omega/10)||1 + j(\omega/100)|} \angle\left(\psi_1 - \beta_1 - \beta_2\right)$$

$$\theta(\omega) = \psi_1 - \beta_1 - \beta_2$$

$$\psi_1 = 90^0$$

$$\beta_1 = \tan^{-1}(\omega/10)$$

$$\beta_2 = \tan^{-1}(\omega/100)$$


100
80
60
40
20
0
-20
-40
-60
-80
-100
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

Compute the phase angle θ(ω) at ω=50, 500, and 1000 rad/s.

$$H(j50) = 0.96\angle -15.25^0 \Rightarrow \theta(j50) = -15.25^0$$

$$H(j500) = 0.22\angle -77.54^0 \Rightarrow \theta(j500) = -77.54^0$$

$$H(j1000) = 0.11\angle -83.72^0 \Rightarrow \theta(j1000) = -83.72^0$$

Compute the steady-state output voltage if the source voltage is given by $v_i(t)=10\cos(500t-25^0)$ V.

$$V_{mo} = |H(j500)|V_{mi} = (0.22)(10) = 2.2V$$

$$\theta_o = \theta(\omega) + \theta_i = -77.54^0 - 25^0 = -102.54^0$$

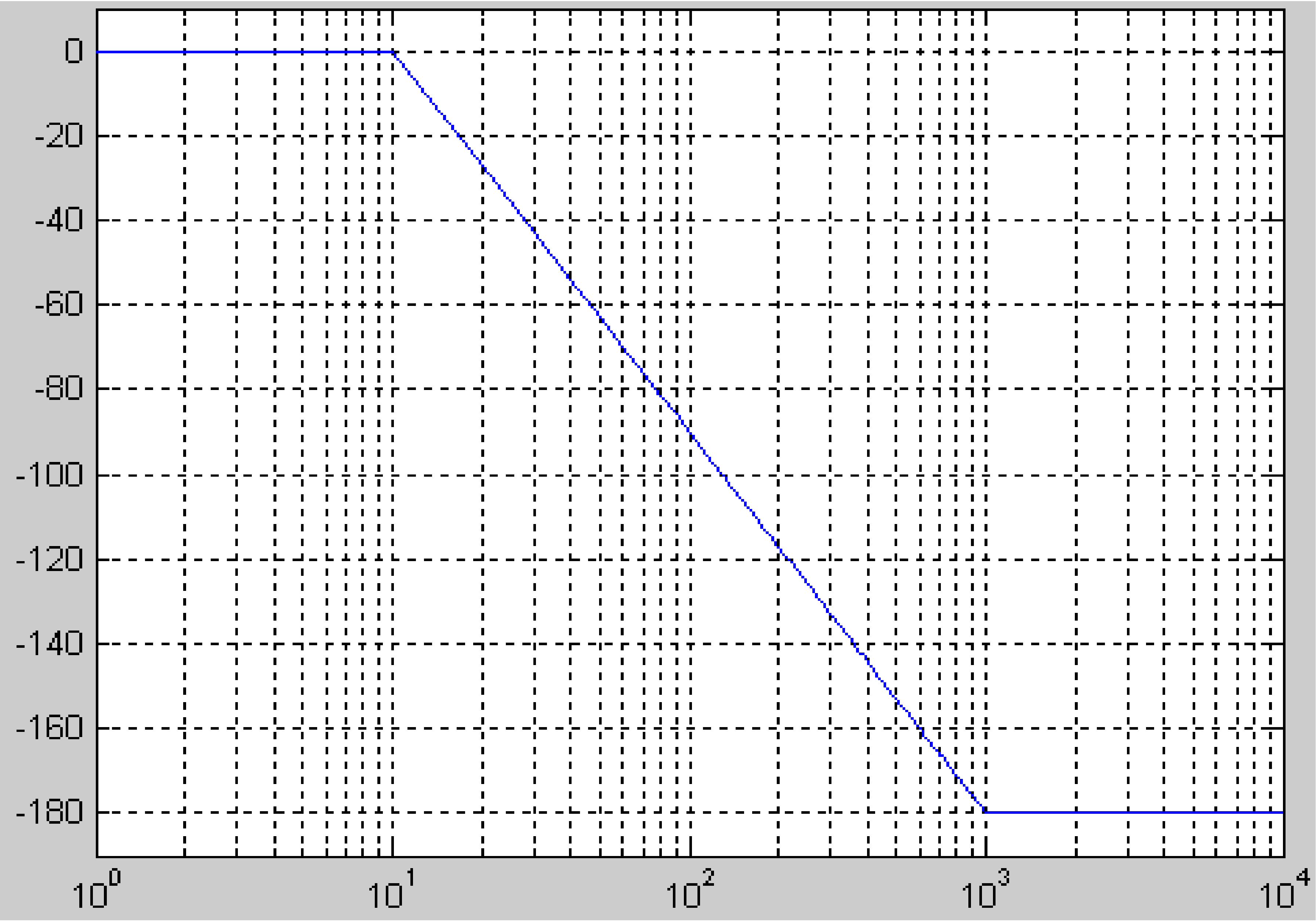
$$v_o(t) = 2.2\cos(500t - 102.54^0)V$$

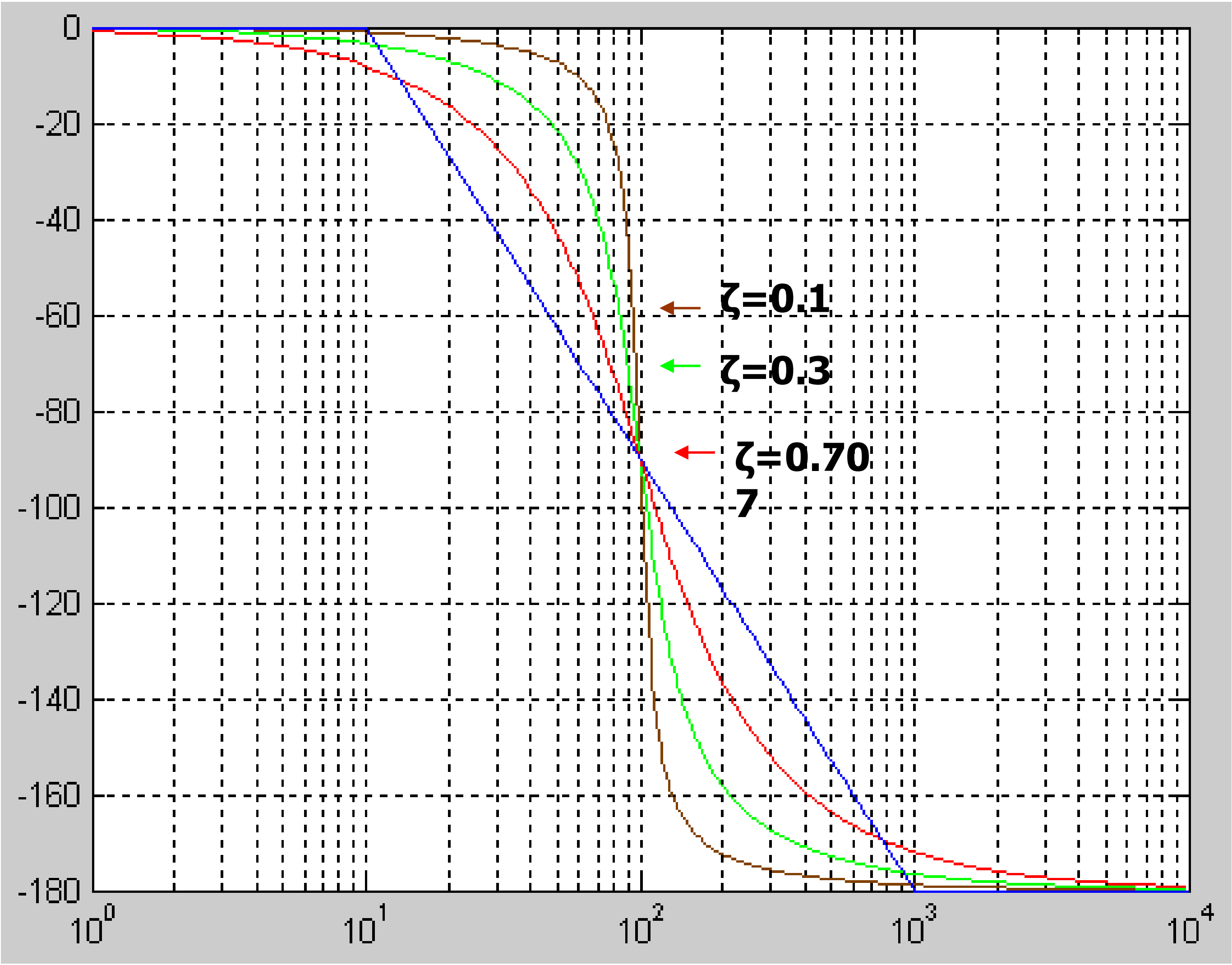
For a second-order zero or pole not at the origin,

For frequencies less than one tenth the corner frequency, the phase angle is assumed to be zero.

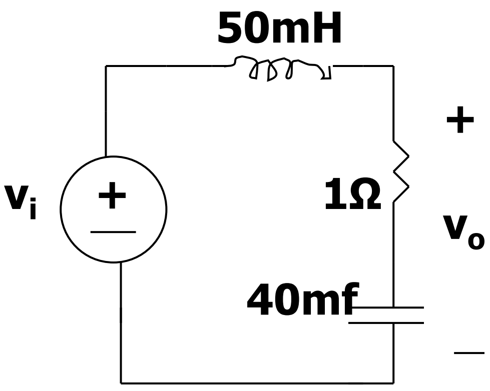
- **For frequencies greater than 10 times the corner frequency, the phase angle is assumed to be ± 180^0.**
- **Between these frequencies the plot is a straight line that goes from 0^0 to ± 180^0 with a slope of ± 90^0/decade.**

As in the case of the amplitude plot, ζ is important in determining the exact shape of the phase angle plot. For small values of ζ, the phase angle changes rapidly in the vicinity of the corner frequency.


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
10^0
10^1
10^2
10^3
10^4


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
10^0
10^1
10^2
10^3
10^4
ζ=0.1
ζ=0.3
ζ=0.70
7

EXAMPLE



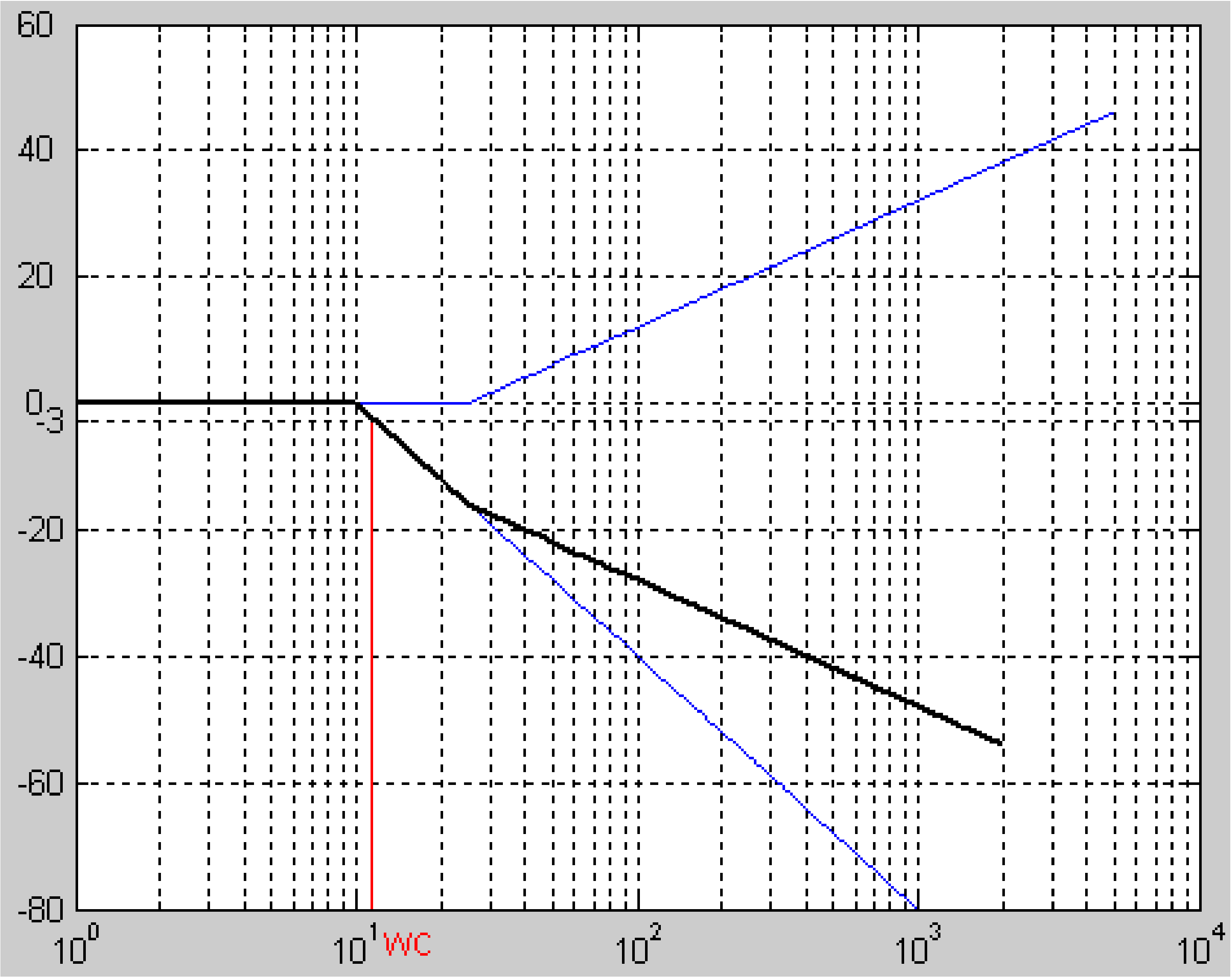
$$H(s)=\frac{\frac{R}{L}s+\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}=\frac{4(s+25)}{s^2+4s+100}$$

$$H(s)=\frac{s/25+1}{1+(s/10)^2+0.4(s/10)}$$

$$H(j\omega)=\frac{\left|1+j\omega/25\right|\angle\psi_1}{\left|1-(\omega/10)^2+j0.4(\omega/10)\right|\angle\beta_1}$$

$$A_{dB}=20\log_{10}\left|1+j\omega/25\right|-20\log_{10}\left|1-(\omega/10)^2+j0.4(\omega/10)\right|$$

$$\theta(\omega)=\psi_1-\beta_1$$



60
40
20
0
-3
-20
-40
-60
-80
10^0
10^1
WC
10^2
10^3
10^4

From the straight-line plot, this circuit acts as a low-pass filter. At the cutoff frequency, the amplitude of H(jω) is 3 dB less than the amplitude in the passband. From the plot, the cutoff frecuency is predicted approximately as 13 rad/s.

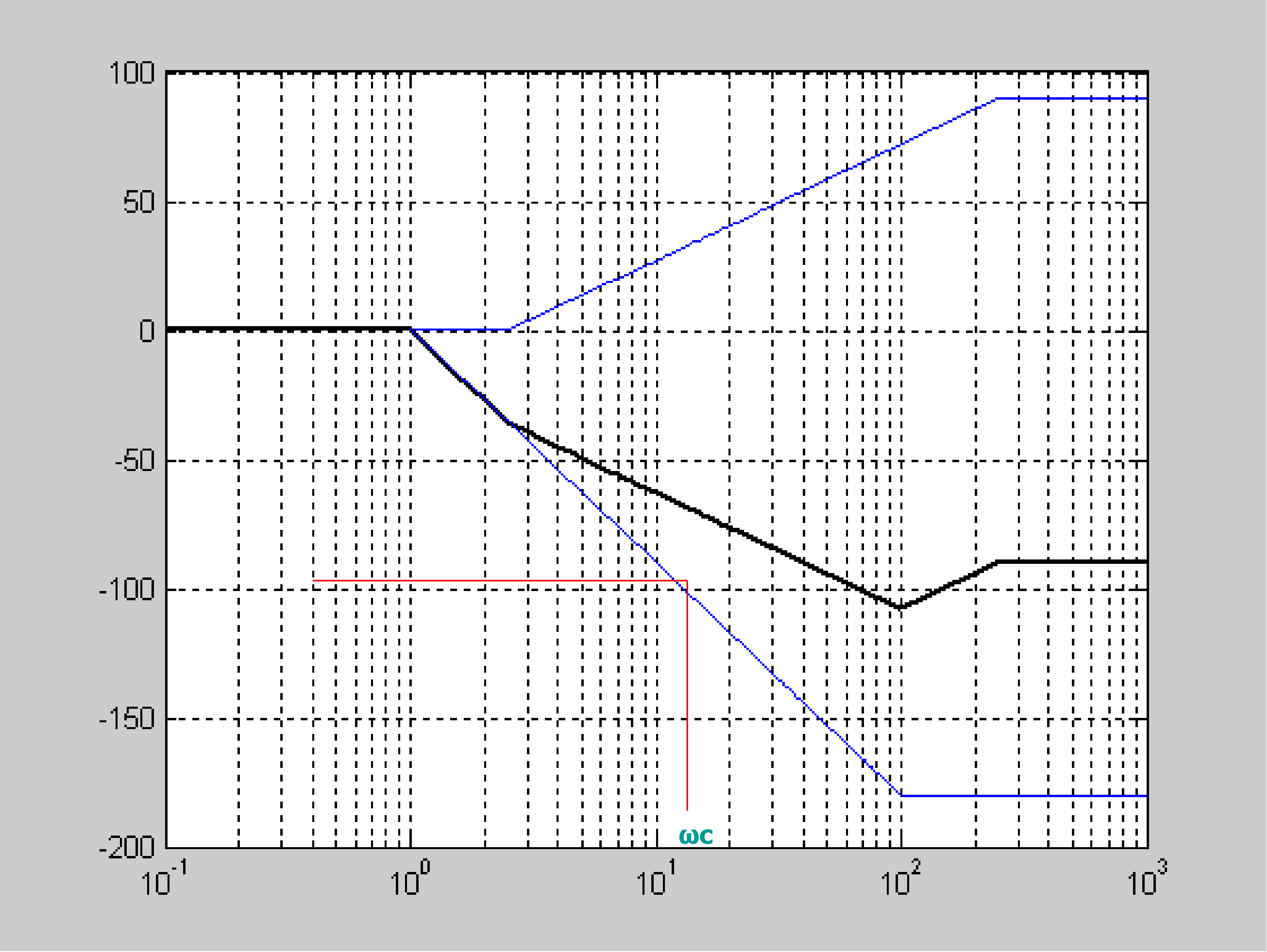
To solve the actual cutoff frequency, follow the procedure as:

$$H_{\max} = 1 \Rightarrow \left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}$$

$$H(j\omega) = \frac{4(j\omega) + 100}{(j\omega)^2 + 4(j\omega) + 100}$$

$$\left|H(j\omega_c)\right| = \frac{\sqrt{(4\omega_c)^2 + 100^2}}{\sqrt{(100 - \omega_c^2)^2 + (4\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = 16 \text{ rad/s}$$



100
50
0
-50
-100
-150
-200
ωc
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

From the phase plot, the phase angle at the cutoff frequency is estimated to be -65^0.

The exact phase angle at the cutoff frequency can be calculated as

$$H(j16) = \frac{4(j16 + 25)}{(j16)^2 + 4(j16) + 100}$$

$$\theta(j16) = \tan^{-1}(16/25) - \tan^{-1}(64/(100 - 16^2)) = -125^0$$

Note the large error in the predicted error. In general, straight-line phase angle plots do not give satisfactory results in the frequency band where the phase angle is changing.

With Our Best Wishes

Automatic Control (2)

Course Staff

Thank You For Your Attention

Mohamed Ahmed Ebrahim